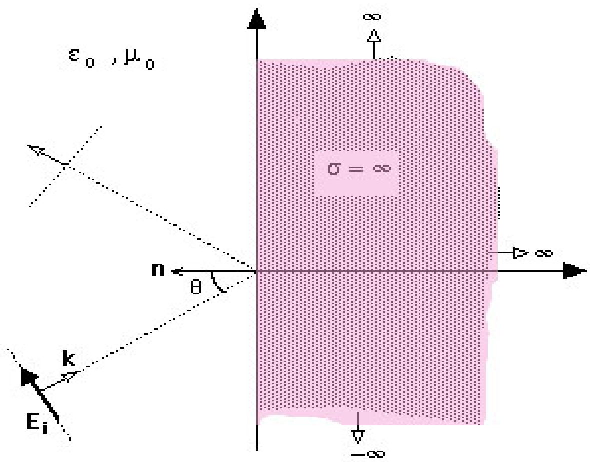
ECECS-611

Reflection from a Metal Surface
Oblique Incidence

A plane wave is incident from left on a perfect conductor at an angle θ with respect to the normal to the surface of the perfect conductor. The electric field is in the plane of incidence.



The incident propagation vector is

$\mathbf{k}_{in} = k_o \sin\theta\, \mathbf{a}_x + k_o \cos\theta\, \mathbf{a}_z$

The reflected propagation vector is

$\mathbf{k}_r = k_o \sin\theta\, \mathbf{a}_x - k_o \cos\theta\, \mathbf{a}_z$

The coordinate vector is

$\mathbf{r} = x\, \mathbf{a}_x + z\, \mathbf{a}_z$

The incident electric field is

$\mathbf{E}_{in} = [\, E_o \cos\theta\, \mathbf{a}_x - E_o \sin \mathbf{a}_z \,]\, e^{-j\,(k_0 \sin\theta\, x + k_o \cos\theta\, z)}$

The reflected Electric field is

$\mathbf{E}_r = [\, \Gamma\, E_o \cos\theta\, \mathbf{a}_x + \Gamma\, E_o \sin \mathbf{a}_z]\, e^{-j\,(k_0 \sin\theta\, x - k_o \cos\theta\, z)}$

The total electric field is

$$\mathbf{E} = [\, E_o \cos\theta\, e^{-j\,(k_0 \sin\theta\, x + k_o \cos\theta\, z)} + \Gamma E_o \cos\theta\, e^{-j\,(k_0 \sin\theta\, x - k_o \cos\theta\, z)} \,]\, \mathbf{a}_x$$
$$+ \; [\, - E_o \sin\theta\, e^{-j\,(k_0 \sin\theta\, x + k_o \cos\theta\, z)} + \Gamma E_o \sin\theta\, e^{-j\,(k_0 \sin\theta\, x - k_o \cos\theta\, z)} \,]\, \mathbf{a}_z$$

The total magnetic field is

$$\mathbf{H} = \frac{E_o}{\eta}\, e^{-j\,(k_0 \sin\theta\, x + k_o \cos\theta\, z)} - \Gamma\, \frac{E_o}{\eta}\, e^{-j\,(k_0 \sin\theta\, x - k_o \cos\theta\, z)}$$

Applying the Boundary condition on the tangential component of the electric field,

$$E_o \cos\theta\, e^{-j\,(k_0 \sin\theta\, x\,)} + \Gamma E_o \cos\theta\, e^{-j\,(k_0 \sin\theta\, x\,)} = 0$$

Simplifying

$$E_o \cos\theta\, (\, 1 + \Gamma\,)\, e^{-j\,(k_0 \sin\theta\, x\,)} = 0 \qquad \Rightarrow \quad \Gamma = -1$$

Submitting this into the x component of E field,

$$\mathbf{E}_x = E_o \cos\theta\, (\, e^{-j\; k_o \cos\theta\, z} - \; e^{\;j\, k_o \cos\theta\, z)})\, e^{-j\,(k_0 \sin\theta\, x\,)}$$
$$= -2j\, E_o \cos\theta\, \sin[(k_o \sin\theta\, z)]\, e^{-j\,(k_0 \sin\theta\, x\,)}$$

Similar result is obtained for the z component. The total magnetic field is

$$\mathbf{H} = \frac{E_o}{\eta} [\, e^{-j\; k_o \cos\theta\, z} + \; e^{j\, k_o \cos\theta\, z)}]\, e^{-j\,(k_0 \sin\theta\, x\,)}\, \mathbf{a}_y$$
$$= 2\, \frac{E_o}{\eta} \cos(k_o \cos\theta\, z)\, e^{-j\,(k_0 \sin\theta\, x\,)}\, \mathbf{a}_y$$

Note that both fields have standing wave properties along the x-axis and propagating properties along the x-axis.

The total electric field electric field which is along the x-axis has zeros every time

$$k_o d \sin\theta \;\; = n\pi.$$

We can then place another metal sheet parallel to the x-axis at a distance

$$d = -\, n\pi/(k_o \sin\theta)$$

from the original metal at z=0 without violating the boundary conditions. Thus we have generated a parallel plate transmission line where the wave propagates along the x-axis with multiple reflections.

This is shown in the figure below.