ECECS-473
SUPPLEMENTARY NOTES

Dissipative Medium

Maxwell's Equations

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = \sigma\mathbf{E} + j\omega\varepsilon\mathbf{E} = (\sigma + j\omega\varepsilon)\mathbf{E}$$

$$\nabla \times (\nabla \times \mathbf{E}) = -j\omega\mu(\nabla \times \mathbf{H})$$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2\mathbf{E} = -j\omega\mu(\sigma + j\omega\varepsilon)\mathbf{E}$$

$$-\nabla^2\mathbf{E} = -j\omega\mu(\sigma + j\omega\varepsilon)\mathbf{E}$$

$$-\nabla^2\mathbf{E} = (-j\omega\mu)(j\omega\varepsilon)\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)\mathbf{E}$$

$$\nabla^2\mathbf{E} + \omega^2\mu\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)\mathbf{E} = 0$$

Definitions

$\frac{\sigma}{\omega\varepsilon} << 1$ Low Loss Dielectric

$$\varepsilon \rightarrow \varepsilon - j\frac{\sigma}{\omega} = \varepsilon' - j\,\varepsilon'' \qquad \varepsilon'' << \varepsilon'$$

$$\tan\delta = \frac{\varepsilon''}{\varepsilon'}$$

ε" =0 Perfect dielectric

$\frac{\sigma}{\omega\varepsilon} >> 1$ Good Conductor

$\sigma = \infty$ Perfect Conductor

Lossy dielectric (low loss)

$$\nabla^2\mathbf{E} + \omega^2\mu(\varepsilon' - j\,\varepsilon'')\mathbf{E} = 0$$

$$jk \rightarrow \gamma = j\omega\sqrt{\mu(\varepsilon' - j\,\varepsilon'')}$$

$$\gamma = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\varepsilon''}{\varepsilon'}} \approx j\omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{1}{2}\tan\delta\right)$$

$$\gamma \approx j\omega\sqrt{\mu\varepsilon'} + \frac{1}{2}\omega\sqrt{\mu\varepsilon'}\tan\delta = (\alpha + j\beta)$$

$$\alpha = \frac{1}{2}\omega\sqrt{\mu\varepsilon'}\tan\delta \qquad \beta = \omega\sqrt{\mu\varepsilon'} \approx k$$

$$\mathbf{E} = \mathbf{E}_o\, e^{-\gamma z} = \mathbf{E}_o\, e^{-(\alpha + j\beta)z} = \mathbf{E}_o\, e^{-\alpha z} e^{-j\beta z}$$

Good Conductor

$$\nabla^2\mathbf{E} + \omega^2\mu\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)\mathbf{E} = 0$$

$$\nabla^2\mathbf{E} - j\,\omega\mu\sigma\,\mathbf{E} = 0$$

$$\gamma^2 = j\,\omega\mu\sigma$$

$$\gamma = \sqrt{j\omega\mu\sigma}$$

$$\sqrt{j} = \sqrt{e^{\,j\frac{\pi}{2}}} = e^{\,j\frac{\pi}{4}} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$\gamma = \alpha + j\beta = \sqrt{\omega\mu\sigma}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$$

$$\mathbf{E} = \mathbf{E}_o\, e^{-\gamma z} = \mathbf{E}_o\, e^{-(\alpha + j\beta)z} = \mathbf{E}_o\, e^{-(z/\delta_s)}\, e^{-j(z/\delta_s)}$$

$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} \quad \text{Skin debth}$$