

# USE OF MATHEMATICS LIBRARY

Roger Chalkley, December 26, 2014

## First search in 1957-1958 of mathematics related to research interests

During the academic year 1957–1958, the Mathematics Department of the College of Arts and Sciences was located in a suite of offices on the east side of McMicken Hall at the top of the main stairwell. That was my only year as an Instructor and I shared my office with Alexander Pyerimhoff, the visiting research professor of mathematics. Counterclockwise from our office were the office of Miss Jean Winston, the office of Albert Barnett, the office of Arno Jaeger, and the shared office of David Lipsich and Gaylord Merriman as department head. The reading room of the Mathematics Research Library was located down one floor from the main entrance of Blegden Library (then, the Main Library) at the opposite end of the hall from the Classics Library.

Valuable advice was given to me by Arno Jaeger in 1957 when he suggested that I make a thorough search of the mathematical literature to find out everything related in any way to the subject of my Ph.D. thesis. That was done by searching through the appropriate pages in each volume of the *Jahrbuch über die Fortschritte der Mathematik* from 1868 through 1938 (the years 1939–1942 were not in our library), each volume of the *Zentralblatt für Mathematik und ihre Grenzgebiete* from 1931 onward, and each volume of *Mathematical Reviews* from 1940 onward. Those review journals were housed in the stack area of Blegden Library. I approached them by walking behind the circulation desk (one floor up from the main entrance) and riding the stack elevator down to the third-floor stack level where the journals were housed. There were tables adjacent to windows on the east side of Blegden's stack-level 3 where the work was done. I found that activity to be fascinating and made notes of numerous results about which I would eventually learn more. That was before photocopiers existed. Altogether, I spent about 4 hours each afternoon for about three weeks on this activity and thoroughly enjoyed it.

The subject of my Ph.D. thesis was the type of ordinary differential equation  $Q = 0$  where  $Q$  is expressible as a quadratic form in the unknown function, its first derivative, and its second derivative with coefficients that can be meromorphic functions. Due to an easily recognizable appearance for  $Q = 0$  when it is given a full representation such as

$$a(z)(y''(z))^2 + b(z)y''(z)y'(z) + c(z)y''(z)y(z) + d(z)(y'(z))^2 + e(z)y'(z)y(z) + f(z)(y(z))^2 = 0,$$

the displayed formulas in the *Fortschritte* and the *Zentralblatt* made review articles related to my subject easy to recognize. Of most importance for my later research, I found that Paul Appell had investigated in 1889 equations of the same type  $Q = 0$  and had obtained very interesting results that I recognized deserved a much deeper study. In particular, he gave conditions on the coefficients of  $Q$  that were necessary and sufficient for  $Q = 0$  to have solutions of the form

$$y(z) = C^2u_1(z) + CKu_2(z) + K^2u_3(z), \quad \text{with } u_1(z), u_2(z), u_3(z) \text{ linearly independent,}$$

where  $C$  and  $K$  are arbitrary constants. My first introduction to the concept of a *relative invariant* occurred when I learned that Paul Appell had found a basic one in 1879 for the equations  $Q = 0$  and that Edmond Laguerre, Georges-Henri Halphen, and Andrew Russell Forsyth had previously found some basic relative invariants for homogeneous linear differential equations. These and other new concepts encountered with this search motivated my principal research interests from 1958 onward.

At that time, almost every page devoted to differential equations in the *Jahrbuch über die Fortschritte der Mathematik* revealed interesting results on subjects that were new

to me. Reports in the *Fortschritte* were carefully written by a fixed group of experts in such a manner that non-specialized mathematicians could acquire a reasonable idea of the subject matter. The reporters for the *Fortschritte* frequently added valuable insights and placed the results in historical perspective. Moreover, they were explicitly critical when that was deserved. Presumably, the *Fortschritte* was terminated after the 1942 volume because, apart from the problems with World War II, the number of mathematical publications of whatever quality were increasing at a rate faster than thoughtful reviews could be prepared. In contrast, articles in *Mathematical Reviews* from 1940 onward and in the *Zentralblatt für Mathematik und ihre Grenzgebiete* have usually been written for specialists by specialists. Sometimes, in the *Zentralblatt*, abstracts written by authors of the publication reviewed were presented as the report. Certainly, the fragmented character of the mathematical literature in modern times was not encouraged by the *Jahrbuch über die Fortschritte der Mathematik*.

The review articles in the *Fortschritte* were so interesting that I supplemented my search by browsing through each section on algebra. Because my first introduction to circulant matrices had occurred just a year earlier in 1956 when Chao-Hui Yang showed me an algebra book of his written in Chinese that explained how the determinant of a  $3 \times 3$  circulant matrix can be used to recall and deduce an identity from which Cardano's formulas are easily derivable, I found it interesting that circulant matrices appeared before 1900 in numerous publications written in French, German, and English. It was later that I learned that circulant matrices should be viewed as a particular type of group matrix. In regard to circulant matrices and group matrices, [click here](#).

### **Second search in early 1980's of mathematics related to research interests**

Around 1960, the Mathematics addition to the east end of the Physics Building was built and the entire structure was later renamed Braunstein Hall. The Mathematics Reading Room in Blegden Library at the opposite end of the hall from the present Classics Reading Room was lost to the Mathematics Department. Various books and journals were transferred from Blegden to a crowded area in what had been the Physics Library at the west end of Braunstein on the top floor. Others books and journals remained behind in Blegden. This scattered situation continued until the early 1970's when the former Chemistry Building, later without malice called the Old Chemistry Building, was remodeled and all of its top floor was prepared for both the rapidly growing Mathematics Department as well as the entire Mathematics Library in an easily accessed location near our offices. In particular, on a balcony of the library in a quiet area up a spiral staircase where there was little distraction, all of the volumes of the *Zentralblatt für Mathematik und ihre Grenzgebiete* from 1931 onward and all the volumes of *Mathematical Reviews* from 1940 onward were readily accessible on shelves adjacent to one long wall parallel to which ten feet away there was a line of tables upon which the reference journals could be studied. Also, all the volumes of the *Jahrbuch über die Fortschritte der Mathematik* for the years 1868 through 1938 (with years 1939, 1940, 1941, 1942 never acquired) were shelved in a safe space easily accessible nearby. Moreover, there was a photocopy machine for use in our department merely a short walk away. That was a wonderful research library and continued that way until 1997.

During the early 1980's, I made a second thorough search through each of the review journals. This involved looking page-by-page through the appropriate sections about differential equations in each volume of the *Jahrbuch über die Fortschritte der Mathematik* from 1868 through 1938 (independently of the 1957–1958 search), each volume of the *Zentralblatt für Mathematik und ihre Grenzgebiete* from 1931 onward, and each volume of *Mathematical Reviews* from 1940 onward. I also read through the sections on linear algebra to compile accurate information about circulant matrices and their influence on later mathematics for reasons explained in the next paragraph. Because the photocopy machine greatly increased

efficiency, this entire operation was also completed in about three weeks with about four hours devoted to it each afternoon. The Inter-Library Loan Service had been organized by then and was extremely helpful. With its aid, I acquired paper copies of the original publications of relevance.

Philip J. Davis kindly mailed to me a copy of his monograph titled *Circulant Matrices* when it was published in 1979 by John Wiley. In a conversation with him, I thanked him for sending it to me and indicated that I truly enjoyed reading it. He then mentioned to me that a paper of mine suggested to him his idea of doing research on the discovery of new results about circulant matrices from largely unexplored viewpoints such as numerical methods and geometry rather than from the standard algebraic viewpoints that had already been heavily explored before the year 1900. That paper of mine was a completely expository one titled *Circulant matrices and algebraic equations* published in *Mathematics Magazine*, Volume 48 (1975), pages 73–80, that focused on some interesting algebraic properties of circulant matrices that were well known before the year 1900 but somewhat obscure to many mathematicians by 1975. Philip J. Davis mentions in his preface to *Circulant Matrices* that: while various algebraic results about circulant matrices are not included in his monograph and no results about circulant matrices as a simple type of group matrix are included in it, their are adequate references for those subjects in his bibliography. The only reference in his bibliography for group matrices is one to a paper of mine titled *Matrices derived from finite abelian groups* that was published in *Mathematics Magazine*, Volume 49 (1976), pages 121–129. Because it was written as an expository paper to be interesting, it made no claims as to who deserves credit for discoveries at particular times. Thus, my search of the mathematical literature in the early 1980's also had the task of acquiring the information needed so that complete details could be given about that if ever required.

I believe the terminology *group matrix* should be restricted to the matrices in the research of Georg Frobenius whose determinants he termed *Gruppendeterminante*. For the many more persons who find the properties of circulant matrices and their natural generalizations to be quite interesting, it seems desirable to use new terminology like *group-pattern matrix* for a matrix defined by **Definition 1** of ([click here, page 2](#)). Moreover, there are numerous patterns of interest for group-pattern matrices. Some for even-order groups can have mirror symmetry with respect to both diagonals as well as rotational symmetry through 180 degrees. In general, circulant matrices merely have symmetry with respect to the secondary diagonal.

### **Third search in mid 1990's of mathematics related to research interests**

My third thorough search of the literature for anything related to my research interests was done during the mid 1990's and emphasized two main topics. After discovering the technique of Lazarus Fuchs for transforming to a simpler form those nonlinear first-order differential equations that uniquely specify a corresponding algebraic curve of genus zero, I was interested in finding anything presented in the literature about that technique. My earlier searches in 1957–1958 and in the early 1980's had uncovered numerous references for later research about relative invariants for various types of differential equations. With this search, I found little of significance for those subjects after the information provided by the *Jahrbuch über die Fortschritte der Mathematik* in its range from 1868 through 1938. Actually, that was good news because the lack of competition meant that I could continue to approach those subjects with thoroughness and the pleasure that provides. The fragmented character of much modern research reflects the need for rapid publication when numerous persons in a particular specialty find some hot topic upon which to swarm.

Again, this search was done by looking page-by-page through the appropriate sections in each volume of the *Jahrbuch über die Fortschritte der Mathematik* from 1868 through 1938,

each volume of the *Zentralblatt für Mathematik und ihre Grenzgebiete* from 1931 onward, and each volume of *Mathematical Reviews* from 1940 onward. Personally, I did not mind reading again various review articles because, with the passage of time, different viewpoints can form and new insights gained. Even with the additional volumes to encounter, this third search was completed in about the same length of time as each of the two previous searches. Namely, four hours each afternoon for about three weeks sufficed. I was able to do each of the three searches during times when my afternoons were free of distractions.

Searches of the literature in the manner just described could be done from the early 1970's until somewhat after Volume 874 as the last paper embodiment of the *Zentralblatt für Mathematik und ihre Grenzgebiete* was acquired in 1997. After that, all the volumes of the *Zentralblatt* were shipped to an inaccessible storage facility and all the Volumes of *Mathematical Reviews* were also shipped to an inaccessible storage facility. The librarians and most of my colleagues seem to regard the on-line review services that replaced them as advantageous for their needs.

### **Concluding observations**

It is clear to me that my research interests and my general perspective about the study of mathematics have both benefited considerably from the particular manner that I was able to use the Mathematics Libraries during the years 1957–1958 and from the early 1970's until the late 1990's. For challenging research topics to pursue whenever time permits, my three library searches of the mathematical literature uncovered a wealth of interesting subjects.