

Francesco Brioschi wrote about relative invariants in 1879 and 1891

As a prominent mathematician, Francesco Brioschi helped attract early attention to relative invariants by publishing two papers on that subject. However, it would be incorrect to rank his work on relative invariants with the significant discoveries made by Edmond Laguerre, Georges-Henri Halphen, Andrew Forsyth, and Paul Appell. An explanation follows.

After Edmond Laguerre obtained in 1879 a relative invariant of weight 3 for homogeneous linear differential equations of order 3, Francesco Brioschi stated in

F. Brioschi, *Sur les équations différentielles linéaires (extrait d'une lettre de M. Brioschi à M. Laguerre)*, Bull. Soc. Math. France **7** (1879), 105–108.

his belief that analogous relative invariants of weight 3 may also exist for homogeneous linear differential equation of order 4, of order 5, etc. Georges-Henri Halphen shortly thereafter obtained explicit formulas for such and was able to establish their invariant properties.

For homogeneous linear differential equations of order $m \geq 4$, it was too tedious to use exact computations for deducing necessary explicit expressions for relative invariants of weight w when $4 \leq w \leq m$. For that reason, Andrew Forsyth introduced in

A. Forsyth, *Invariants, covariants, and quotient derivatives associated with linear differential equations*, Philosophical Transactions of the Royal Society of London **179** (1888), 377–489.

the use of infinitesimal transformations to simplify computations by omitting the squares, cubes, . . . of various infinitesimals. In that manner, he deduced expressions corresponding to weights w that satisfy $3 \leq w \leq 7$ and $w \leq m$ for homogeneous linear differential equations of order m . Shortly thereafter, Francisco Brioschi also assumed in

F. Brioschi, *Les invariants des équations différentielles linéaires*, Acta. Mat. **14**, (1891), 233–248.

the existence of basic relative invariants of weight w when $3 \leq w \leq m$ for homogeneous linear differential equations of order m and made several conclusions. However, his main feature was the presentation of formulas that would be necessary for relative invariants to have when their weights satisfy $w = 3 \leq m$, $w = 4 \leq m$, $w = 5 \leq m$, $w = 6 \leq m$, and $w = 7 \leq m$. In particular, because he rewrote for that purpose the formulas Forsyth had deduced using infinitesimal transformations and because Forsyth's formula for the case $w = 7 \leq m$ has a single error where a denominator is written as $22(m + 1)^2$ in place of $11(m + 1)^2$, Brioschi's formula for the case $w = 7 \leq m$ introduced additional errors. For details, see page 79 of

R. Chalkley, *Basic Global Relative Invariants for Homogeneous Linear Differential Equations*, Memoirs Amer. Math. Soc. **156** (2002), Number 744.

In particular, the errors that Brioschi introduced were copied exactly into the formula for the case $w = 7 \leq m$ that appear on page 196 of

L. Schlesinger, *Handbuch der Theorie der linearen Differentialgleichungen, Band II, Teil 1*, Teubner, Leipzig, 1897.