

Avoid a Progress-Hindering Blunder

Roger Chalkley, October 22, 2018

The specific examples of invariants for transformations of homogeneous linear differential equations provided by James Cockle, Edmund Laguerre, and George-Henri Halphen during the years 1862–1884 produced a splendid optimism about how their results might be extended to homogeneous linear differential equations of any order. However, that optimism gradually faded as numerous research papers and monographs were written to illustrate and advance the idea that further progress depended upon presenting the subject as a special detail in the application of infinitesimal transformations. Because that viewpoint became predominant and produced nothing of significance, the subject gradually withered until 1989 when it became clear to us that:

research on the subject before 1989 had reached a thoroughly confused state that required a complete redevelopment for progress to be made.

The most serious defect was that no one prior to 1989 had published satisfactory formulas for the coefficients of m th-order homogeneous linear differential equations that result from changes of the independent variable where m remains a symbol representing any positive integer. That key shortcoming is described in Chapter 15 of my monograph titled *The Research about Invariants of Ordinary Differential Equations*; for that, [click here](#). However, contributing to that was an inappropriate notation where:

all persons who published papers before 1989 about invariants for m th-order homogeneous linear differential equations needlessly and unquestioningly inserted binomial coefficients in each of their representations for such equations.

Thus, apart from the symbolism z and $y(z)$, all of the monic homogeneous linear differential equation considered for transformations before 1989 were written like

$$(1) \quad y^{(m)}(z) + \sum_{i=1}^m \binom{m}{i} C_i(z) y^{(m-i)}(z) = 0, \quad \text{with} \quad \binom{m}{i} = \frac{m!}{i!(m-i)!}.$$

For a clear explanation about why the binomial notation as in (1) should never have been employed, see page 1 as well as Chapters 15 and 18 of my monograph titled *The Research about Invariants of Ordinary Differential Equations*. In particular, for page 1 [click here](#), for Chapter 15 [click here](#), and for Chapter 18 [click here](#).