The Conditional Central Limit Question For Superlinear Processes

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Let $\mathbb{Z}$ denote the integers and J a countable set; let $F_j,\ j \in J$, be distribution functions with means 0 and unit variances; let $\xi_{i,j},\ i \in \mathbb{Z},\ j \in J$, be independent random variables defined on a probability space $(\Omega, \mathcal{A}, P)$; and suppose $\xi_{i,j} \sim F_j$ are identically distributed for each j. Next let $c_{i,j}$ be a square summable array and consider processes of the form

$$X_k = \sum_{j \in J} \sum_{i=0}^{\infty} c_{i,j} \xi_{k-i,j} = \sum_{j \in J} \sum_{i \leq k} c_{k-i,j} \xi_{i,j}. \tag{1}$$

Here X_k converges *w.p.1* and in $L^2(P)$ by the Three Series Theorem and defines a strictly stationary process. We will call X_k a *superlinear process*, as it is the sum (superposition) of linear processes. Let $\mathcal{F}_0$ be the sigma algebra generated by $\xi_{i,j},\ i \leq 0,\ j \in J$, $S_n = X_1 + \cdots + X_n,\ \sigma_n^2 = E(S_n^2)$, suppose that $\sigma_n \to \infty$, and let F_n be a regular conditional distribution function for S_n/σ_n given $\mathcal{F}_0$,

$$F_n(\omega; z) = P[\frac{S_n}{\sigma_n} \leq z | \mathcal{F}_0](\omega).$$

Necessary and sufficient conditions are given for the convergence in probability of F_n to the standard normal distribution Φ. The conditions resemble the Lindeberg Feller Conditions and involve an interesting interplay between the coefficients $c_{i,j}$ and the distribution functions F_j.