

**Rick Bradley**

**A strictly stationary,  $N$ -tuplewise independent counterexample to the central limit theorem (joint work with Alexander Pruss)**

This is joint work with Alexander Pruss. For any given integer  $N \geq 2$ , there exists a strictly stationary, ergodic sequence  $(X_k, k \in \mathbf{Z})$  of random variables with the following properties: (i) the random variable  $X_0$  is uniformly distributed on the interval  $[-3^{1/2}, 3^{1/2}]$  (and hence  $EX_0 = 0$  and  $EX_0^2 = 1$ ); (ii) for every choice of  $N$  distinct integers  $k(1), k(2), \dots, k(N)$ , the random variables  $X_{k(1)}, X_{k(2)}, \dots, X_{k(N)}$  are independent; (iii) the random variables  $|X_k|, k \in \mathbf{Z}$  are independent; and (iv) for every infinite set  $Q \subset \mathbf{N}$ , there exist an infinite set  $T \subset Q$  and a nondegenerate, non-normal probability measure  $\mu$  on  $\mathbf{R}$  such that  $(X_1 + X_2 + \dots + X_n)/n^{1/2}$  converges to  $\mu$  in distribution as  $n \rightarrow \infty, n \in T$ . This example is a modification of a somewhat similar, nonstationary,  $N$ -tuplewise independent, identically distributed counterexample in Pruss [PTRF 111 (1998) 323-332]. It complements the strictly stationary, pairwise independent counterexamples in Janson [Stochastics 23 (1988) 439-448], and the strictly stationary, triplewise independent counterexamples developed in Bradley [PTRF 81 (1989) 1-10 and Rocky Mountain J. Math. 37 (2007) 25-44].