

Exercises for applications of matrix diagonalization

1. Use matrix diagonalization to solve the recursion

$$y_{n+1} = 5y_n - 6y_{n-1}, \quad y_0 = 2, y_1 = 5$$

Answer: $y_n = 2^n + 3^n$

2. Use matrix diagonalization to solve the differential equation

$$y'' = 5y' - 6y, \quad y(0) = 2, y'(0) = 5$$

Answer: $y(t) = e^{2t} + e^{3t}$

3. Use matrix diagonalization to solve the vector recursion

$$\begin{aligned} x_{n+1} &= -x_n + y_n \\ y_{n+1} &= x_n - y_n \\ x_0 &= 0 \\ y_0 &= 2 \end{aligned}$$

Answer: for $n > 0$, $x_n = -(-2)^n$, $y_n = (-2)^n$

4. Use matrix diagonalization to solve the system of differential equations

$$\begin{aligned} u' &= -u + v \\ v' &= u - v \\ u(0) &= 0 \\ v(0) &= 2 \end{aligned}$$

Answer: $u(t) = 1 - e^{-2t}$, $v(t) = 1 + e^{-2t}$