Dynamical Systems Quiz-8 Key A

Instructions. Be sure to indicate how you got your answers.

1. Analyze equation $x' = (x^2-1)(x+2)$

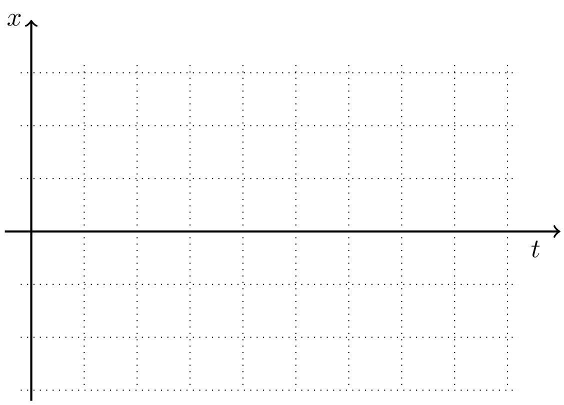
Hint: $(x^2-1)(x+2) = (x-1)(x+1)(x+2) = x^3+2x^2-x-2$

(a) Determine all equilibria and their linearized stability.

This is autonomous equation $y' = f(x)$ with equilibria at the roots of $f(x) = 0$. So the equilibria are $a = -2, -1, 1$.

a	$f'(a)$	type
-2	$3 > 0$	unstable
-1	$-2 < 0$	stable
1	$6 > 0$	unstable

(b) Sketch some time-dependent solutions based on the one-dimensional phase line.



(c) Use your stability analysis to determine $\lim_{t\to\infty} x(t)$ for the solution with the initial value $x(0) = 1/2$.

This solution converges to the closest stable equilibrium $\lim_{t\to\infty} x(t) = -1$

Dynamical Systems Quiz-8 Key B

Instructions. Be sure to indicate how you got your answers.

1. Analyze equation $x' = (x^2 - 4)(x + 1)$

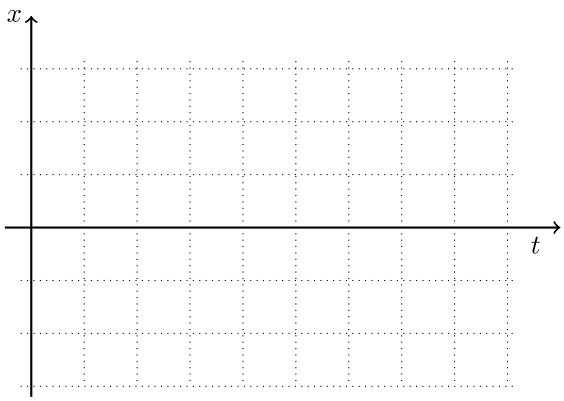
   Hint: $(x^2 - 4)(x + 1) = (x - 2)(x + 2)(x + 1) = x^3 + x^2 - 4x - 4$

   (a) Determine all equilibria and their linearized stability.

   This is autonomous equation $y' = f(x)$ with equilibria at the roots of $f(x) = 0$. So the equilibria are $a = -2, -1, 1$.

a	$f'(a)$	type
-2	$4 > 0$	unstable
-1	$-3 < 0$	stable
2	$12 > 0$	unstable

   (b) Sketch some time-dependent solutions based on the one-dimensional phase line.



   (c) Use your stability analysis to determine $\lim_{t\to\infty} x(t)$ for the solution with initial value $x(0) = 3/2$.

   This solution converges to the closest stable equilibrium $\lim_{t\to\infty} x(t) = -1$

 Name ____________________

Dynamical Systems Quiz-8 Key C

Instructions. Be sure to indicate how you got your answers.

1. Analyze equation $x' = (4 - x^2)(x - 1)$

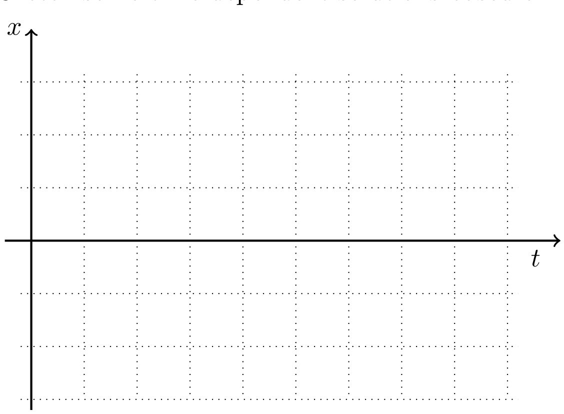
   Hint: $(x^2 - 1)(x + 2) = (x - 1)(x + 2)(2 - x) = -x^3 + x^2 + 4x - 4$

   (a) Determine all equilibria and their linearized stability.

   This is autonomous equation $y' = f(x)$ with equilibria at the roots of $f(x) = 0$. So the equilibria are $a = -2, -1, 1$.

a	$f'(z)$	type
-2	-12	stable
1	$3 > 0$	unstable
2	$-4 < 0$	stable

   (b) Sketch some time-dependent solutions based on the one-dimensional phase line.



   (c) Use your stability analysis to determine $\lim_{t\to\infty} x(t)$ for the solution with initial value $x(0) = 1/2$.

   This solution converges to the closest stable equilibrium $\lim_{t\to\infty} x(t) = -2$