Dynamical Systems Quiz-7 Key A

Instructions. Vectors and matrices are typeset in bold. Simplify your answers when appropriate. Be sure to indicate how you got your answers.

1. Write the system of equations

$$\begin{aligned}\frac{dx}{dt} &= 2x + 3y \\ \frac{dy}{dt} &= 7y\end{aligned}$$

in matrix form $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$ by determining matrix $\mathbf{A}$. Answer: $\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & 7 \end{bmatrix} \vec{x}$

2. Given that matrix $\mathbf{A}$ has the following eigenvalues/eigenvectors $\lambda_1 = 2, \vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\lambda_2 = -3, \vec{v}_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ is the general solution of $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$, determine the formula for $\vec{x}$ and for the second component y of the solution.
Answer: $\vec{x} = c_1 e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ so $y = c_1 e^{2t} - 2c_2 e^{-3t}$

3. Determine the eigenvalues and the eigenvectors for matrix $\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$. Answer: $\lambda_1 = 3$ with $\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\lambda_2 = -2$ with $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Dynamical Systems Quiz-7 Key B

Instructions. Vectors and matrices are typeset in bold. Simplify your answers when appropriate. Be sure to indicate how you got your answers.

1. Write the system of equations

$$\begin{aligned} \frac{dx}{dt} &= 3x + 2y \\ \frac{dy}{dt} &= 7y \end{aligned}$$

in matrix form $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$ by determining matrix $\mathbf{A}$. Answer: $\frac{d\vec{x}}{dt} = \begin{bmatrix} 3 & 2 \\ 0 & 7 \end{bmatrix} \vec{x}$

2. Given that matrix $\mathbf{A}$ has the following eigenvalues/eigenvectors $\lambda_1 = -2, \vec{v}_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\lambda_2 = 3, \vec{v}_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ is the general solution of $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$, determine the formula for $\vec{x}$ and for the second component y of the solution.
Answer: $\vec{x} = c_1 e^{-2t} \begin{bmatrix} -2 \\ 3 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ so $y = 3c_1 e^{-2t} + 2c_2 e^{3t}$

3. Determine the eigenvalues and the eigenvectors for matrix $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 5 & 0 \end{bmatrix}$. Answer: $\lambda_1 = 5$ with $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\lambda_2 = -3$ with $\vec{v}_2 = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$

 Name ____________________

Dynamical Systems Quiz-7 Key C

Instructions. Vectors and matrices are typeset in bold. Simplify your answers when appropriate. Be sure to indicate how you got your answers.

1. Write the system of equations

$$\begin{aligned}\frac{dx}{dt} &= 3x + 7y \\ \frac{dy}{dt} &= 2y\end{aligned}$$

in matrix form $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$ by determining matrix $\mathbf{A}$. Answer: $\frac{d\vec{x}}{dt} = \begin{bmatrix} 3 & 7 \\ 0 & 2 \end{bmatrix} \vec{x}$

2. Given that matrix $\mathbf{A}$ has the following eigenvalues/eigenvectors $\lambda_1 = 5, \vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\lambda_2 = -2, \vec{v}_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ is the general solution of $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$, determine the formula for $\vec{x}$ and for the second component y of the solution.
Answer: $\vec{x} = c_1 e^{5t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ so $y = c_1 e^{5t} - 2c_2 e^{-2t}$

3. Determine the eigenvalues and the eigenvectors for matrix $\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$. Answer: $\lambda_1 = 3$ with $\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\lambda_2 = -2$ with $\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$