Printed: February 17, 2023 Name ____________________

Dynamical Systems Quiz-5 Key

Instructions. Simplify your answers when appropriate. Be sure to indicate how you got your answers.

1. Find the Laplace transforms of

   (a) $f(t) = e^{3t+1}$ Hint: use linearity of $\mathcal{L}$ Answer: $\boxed{F(s) = \frac{e}{s-3}}$

   (b) $f(t) = \begin{cases} 1 & 1 < t < 2 \\ 0 & \text{otherwise} \end{cases}$ Hint: compute the integral $F(s) = \int_1^2 e^{-st}dt = \boxed{\frac{e^{-s}-e^{-2s}}{s}}$

   (c) $f(t) = \cos^2(2t)$. Hint: rewrite f using double angle formula $f(t) = \frac{1}{2}\cos 4t + \frac{1}{2}$ so $\boxed{F(s) = \frac{s/2}{s^2+16} + \frac{1}{2s}}$

2. Determine which function has Laplace transform

   (a) $F(s) = \frac{2}{s-1} - \frac{3}{s-2}$ Hint: use the table Answer: $\boxed{f(t) = 2e^t - 3e^{2t}}$

   (b) $F(s) = \frac{3s+1}{s^2+4}$ Hint: split into sum and use the table Answer: $\boxed{f(t) = 3\cos 2t + \frac{1}{2}\sin 2t}$

   (c) $F(s) = \frac{9+s}{4-s^2}$ Hint: use partial fractions and the table $F(s) = \frac{7}{4(s+2)} - \frac{11}{4(s-2)}$ so $\boxed{f(t) = \frac{7}{4}e^{-2t} - \frac{11}{4}e^{2t}}$

   (d) $F(s) = \frac{2s-3}{s^2-s-6}$ Hint: use partial fractions $F(s) = \frac{7}{5(s+2)} + \frac{3}{5(s-3)}$ so $\boxed{f(t) = \frac{3}{5}e^{3t} + \frac{7}{5}e^{-2t}}$

Table of Laplace transforms $\mathcal{L}(f) = \int_0^\infty e^{-ts} f(t) dt$

$f(t)$	$F(s) = \mathcal{L}(f)$	(domain of $F(s)$)
1	$\frac{1}{s}$	$(s > 0)$
t^n	$\frac{n!}{s^{n+1}}$	$(s > 0)$
e^{at}	$\frac{1}{s-a}$	$(s > a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$(s > 0)$
$\sin at$	$\frac{a}{s^2+a^2}$	$(s > 0)$
$H(t-c)$	$\frac{e^{-cs}}{s}$	$(s > 0)$
$\delta(t-a)$	e^{-as}	$(s > 0)$

Laplace transform formulas

$$\begin{aligned}
\mathcal{L}(f_1(t) + c f_2(t)) &= \mathcal{L}(f_1(t)) + c\mathcal{L}(f_2(t)) \\
\mathcal{L}\left(f^{(n)}(t)\right) &= s^n F(s) - s^{n-1} f(0) - \cdots - s f^{(n-2)}(0) - f^{(n-1)}(0) \\
\mathcal{L}\left(e^{at} f(t)\right) &= F(s-a) \\
\mathcal{L}(t^k f(t)) &= (-1)^k \frac{d^k}{ds^k} F(s) \\
\mathcal{L}\left(H(t-c) f(t-c)\right) &= e^{-cs} F(s) \\
\mathcal{L}\left(\int_o^t f(\tau) g(t-\tau)\, d\tau\right) &= F(s) \cdot G(s) \\
\mathcal{L}\left(f(at)\right) &= \frac{1}{a} F\left(\frac{s}{a}\right)
\end{aligned}$$