Printed: February 3, 2023 Name ______________________________

Dynamical Systems Quiz-4

A Key

Instructions. Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Solve the initial value problem $x'' - 4x = 6e^t$, $x(0) = 7$, $x'(0) = 0$.

   The homogeneous equation $x'' - 4x = 0$ has characteristic equation $r^2 - 4 = 0$ with roots $r = \pm 2$ that give $x_1 = e^{2t}, x_2 = e^{-2t}$.

   Using the method of undetermined coefficients, we seek a particular solution $x_p = Ae^t$. We compute $A = -2$.

   So the formula $x = C_1x_1 + C_2x_2 + x_p$ for the general solution gives $x(t) = C_1e^{2t} + C_2e^{-2t} - 2e^t$.

   Using the initial condition we now compute

$$\begin{aligned} x(0) = C_1 + C_2 - 2 &= 7 \\ x'(0) = 2C_1 - 2C_2 - 2 &= 0 \end{aligned}$$

   We get $C_1 = 5, C_2 = 4$.

   The answer is $\boxed{x(t) = 5e^{2t} + 4e^{-2t} - 2e^t}$

Turn the page for question 2

2. Find the general solution of the nonhomogeneous equation $x'' - 4x' + 4x = 6t$.

   We first solve the homogeneous equation $x'' - 4x' + 4x = 0$. The characteristic equation is $r^2 - 4r + 4 = (r-2)^2 = 0$, with double root. The general solution of the homogeneous equation is $x = C_1e^{2t} + C_2te^{2t}$.

   By the method of undetermined coefficients, we seek particular solution of the form $x_p = A + Bt$. We get $B = \frac{3}{2}$, $A = B = \frac{3}{2}$ so $x_p = \frac{3}{2}(1+t)$.

   Combining the two parts together, we get the answer: $\boxed{x = C_1e^{2t} + C_2te^{2t} + \frac{3}{2}(1+t)}$.

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Dynamical Systems Quiz-4

B Key

Instructions. Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Solve the initial value problem $x'' - 4x = 18e^t$, $x(0) = 5$, $x'(0) = 0$.

   The homogeneous equation $x'' - 4x = 0$ has characteristic equation $r^2 - 4 = 0$ with roots $r = \pm 2$ that give $x_1 = e^{2t}, x_2 = e^{-2t}$.

   Using the method of undetermined coefficients, we seek a particular solution $x_p = Ae^t$. We compute $A = -6$.

   So the formula $x = C_1x_1 + C_2x_2 + x_p$ for the general solution gives $x(t) = C_1e^{2t} + C_2e^{-2t} - 6e^t$.

   Using the initial condition we now compute

$$\begin{aligned} x(0) = C_1 + C_2 - 6 &= 5 \\ x'(0) = 2C_1 - 2C_2 - 6 &= 0 \end{aligned}$$

   We get $C_1 = 7, C_2 = 4$.

   The answer is $\boxed{x(t) = 7e^{2t} + 4e^{-2t} - 6e^t}$

Turn the page for question 2

2. Find the general solution of the nonhomogeneous equation $x'' - 4x' + 4x = 10t$.

   We first solve the homogeneous equation $x'' - 4x' + 4x = 0$. The characteristic equation is $r^2 - 4r + 4 = (r-2)^2 = 0$, with double root. The general solution of the homogeneous equation is $x = C_1e^{2t} + C_2te^{2t}$.

   By the method of undetermined coefficients, we seek particular solution of the form $x_p = A + Bt$. We get $B = \frac{5}{2}$, $A = B = \frac{5}{2}$ so $x_p = \frac{5}{2}(1+t)$.

   Combining the two parts together, we get the answer: $\boxed{x = C_1e^{2t} + C_2te^{2t} + \frac{5}{2}(1+t)}$.

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Dynamical Systems Quiz-4

C Key

Instructions. Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Solve the initial value problem $x'' - 4x = 12e^t$, $x(0) = 2$, $x'(0) = 0$.

   The homogeneous equation $x'' - 4x = 0$ has characteristic equation $r^2 - 4 = 0$ with roots $r = \pm 2$ that give $x_1 = e^{2t}, x_2 = e^{-2t}$.

   Using the method of undetermined coefficients, we seek a particular solution $x_p = Ae^t$. We compute $A = -4$.

   So the formula $x = C_1x_1 + C_2x_2 + x_p$ for the general solution gives $x(t) = C_1e^{2t} + C_2e^{-2t} - 4e^t$.

   Using the initial condition we now compute

$$\begin{aligned} x(0) = C_1 + C_2 - 4 &= 2 \\ x'(0) = 2C_1 - 2C_2 - 4 &= 0 \end{aligned}$$

   We get $C_1 = 4, C_2 = 2$.

   The answer is $\boxed{x(t) = 4e^{2t} + 2e^{-2t} - 4e^t}$

Turn the page for question 2

2. Find the general solution of the nonhomogeneous equation $x'' - 4x' + 4x = 8t$.

   We first solve the homogeneous equation $x'' - 4x' + 4x = 0$. The characteristic equation is $r^2 - 4r + 4 = (r-2)^2 = 0$, with double root. The general solution of the homogeneous equation is $x = C_1e^{2t} + C_2te^{2t}$.

   By the method of undetermined coefficients, we seek particular solution of the form $x_p = A + Bt$. We get $B = 2$, $A = B = 2$ so $x_p = 2(1 + t)$.

   Combining the two parts together, we get the answer: $\boxed{x = C_1e^{2t} + C_2te^{2t} + 2(1 + t)}$.

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Dynamical Systems Quiz-4

D Key

Instructions. Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Solve the initial value problem $x'' - 4x = 12e^t$, $x(0) = 4$, $x'(0) = 0$.

   The homogeneous equation $x'' - 4x = 0$ has characteristic equation $r^2 - 4 = 0$ with roots $r = \pm 2$ that give $x_1 = e^{2t}, x_2 = e^{-2t}$.

   Using the method of undetermined coefficients, we seek a particular solution $x_p = Ae^t$. We compute $A = -4$.

   So the formula $x = C_1 x_1 + C_2 x_2 + x_p$ for the general solution gives $x(t) = C_1 e^{2t} + C_2 e^{-2t} - 4e^t$.

   Using the initial condition we now compute

$$\begin{aligned} x(0) = C_1 + C_2 - 4 &= 4 \\ x'(0) = 2C_1 - 2C_2 - 4 &= 0 \end{aligned}$$

   We get $C_1 = 5, C_2 = 3$.

   The answer is $\boxed{x(t) = 5e^{2t} + 3e^{-2t} - 4e^t}$

Turn the page for question 2

2. Find the general solution of the nonhomogeneous equation $x'' - 4x' + 4x = 5t$.

   We first solve the homogeneous equation $x'' - 4x' + 4x = 0$. The characteristic equation is $r^2 - 4r + 4 = (r-2)^2 = 0$, with double root. The general solution of the homogeneous equation is $x = C_1 e^{2t} + C_2 t e^{2t}$.

   By the method of undetermined coefficients, we seek particular solution of the form $x_p = A + Bt$. We get $B = \frac{5}{4}$, $A = B = \frac{5}{4}$ so $x_p = \frac{5}{4}(1 + t)$.

   Combining the two parts together, we get the answer: $\boxed{x = C_1 e^{2t} + C_2 t e^{2t} + \frac{5}{4}(1 + t)}$.