Dynamical Systems Quiz-2

A Key

Instructions. Be sure to show your work so that it is clear how you got your answers. Simplify the answers!

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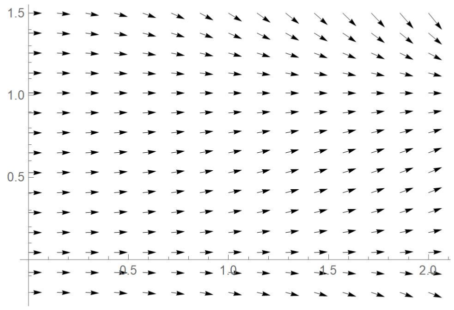
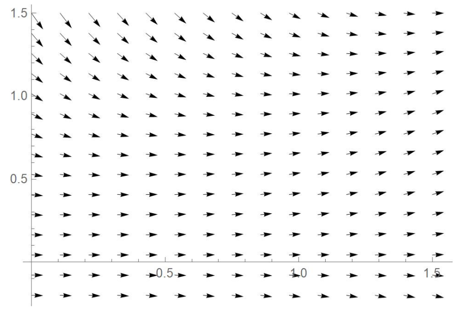
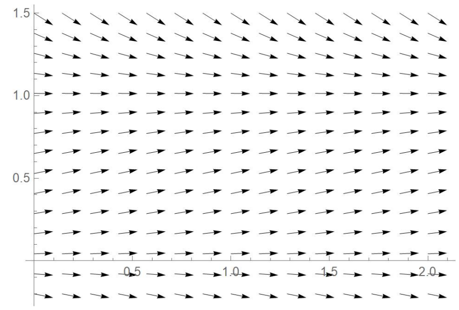
1. Match the direction fields with the differential equations: (use arrows to indicate your matches)



This is $x' = tx(1-x)$



This is $x' = x(t-x)$



This is $x' = x(1-x)$

$\boxed{x' = x(1-x)}$ $\quad$ $\boxed{x' = tx(1-x)}$ $\quad$ $\boxed{x' = x(t-x)}$

2. Solve the initial value problem using separation. Give explicit solution.

$$\frac{dx}{dt} + 12tx = 0, \quad x(0) = -1$$

$\int \frac{dx}{x} = -12 \int t dt$ so $\ln|x| = -6t^2 + c$ or $x = Ce^{-6t^2}$.

$\boxed{x(t) = -e^{-6t^2}}$

(turn the page for question 3)

3. Solve $\frac{dx}{dt} + x = t$, $x(0) = -2$

Using formula, the integrating factor is $\mu(t) = e^{\int 1 dt} = e^t$, so the solution is $x = \frac{1}{\mu(t)} \int t\mu(t)dt + c/\mu(t) = e^{-t} \int te^t dt + ce^{-t} = e^{-t}(te^t - e^t) + ce^{-t}$ where we integrated by part $\in uv' = uv - \int u'v$.

This simplifies to $x(t) = t - 1 + ce^{-t}$. Computing c from the initial condition, we get

$\boxed{x(t) = t - 1 - e^{-t}}$

Printed: January 20, 2023 Name ______________________

Dynamical Systems Quiz-2

B Key

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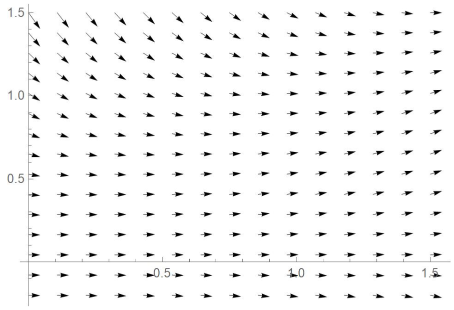
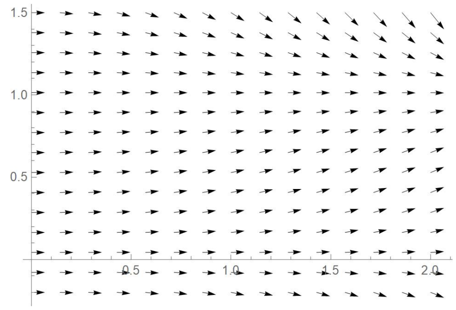
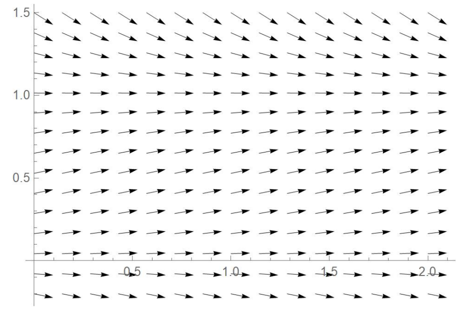
1. Match the direction fields with the differential equations: (use arrows to indicate your matches)



This is $x' = x(t - x)$



This is $x' = tx(1 - x)$



This is $x' = x(1 - x)$

$\boxed{x' = x(1-x)}$ $\boxed{x' = tx(1-x)}$ $\boxed{x' = x(t-x)}$

2. Solve the initial value problem using separation. Give explicit solution.

$$\frac{dx}{dt} + 6tx = 0, \quad x(0) = -2$$

$\int \frac{dx}{x} = -6 \int t dt$ so $\ln |x| = -3t^2 + c$ or $x = Ce^{-3t^2}$.

$\boxed{x(t) = -2e^{-3t^2}}$

(turn the page for question 3)

3. Solve $\frac{dx}{dt} + x = t$, $x(0) = 2$

Here is a different solution, based on Section 1.7. The general solution of the homogeneous equation $\frac{dx}{dt} + x = 0$ is $x_h = ce^{-t}$. A particular solution can be guessed as a polynomial $x_p = A + Bt$. Putting this into the non-homogeneous equation we get $B + (A + Bt) = t$ so $B = 1$ and $A = -1$. The general solution is $x = x_h + x_p = ce^{-t} + t - 1$. We determine the constant from the initial condition:

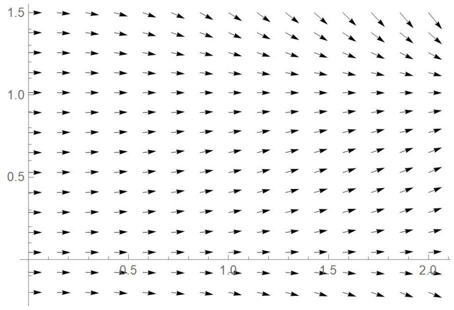
$\boxed{x(t) = t - 1 + 3e^{-t}}$

Dynamical Systems Quiz-2

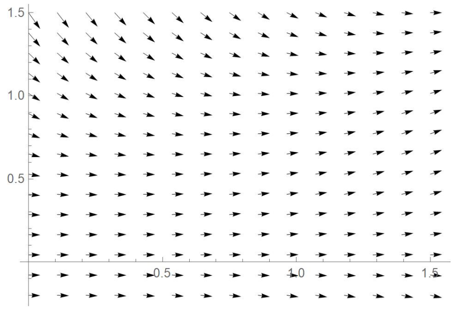
C Key

Instructions. Be sure to show your work so that it is clear how you got your answers. Simplify the answers!

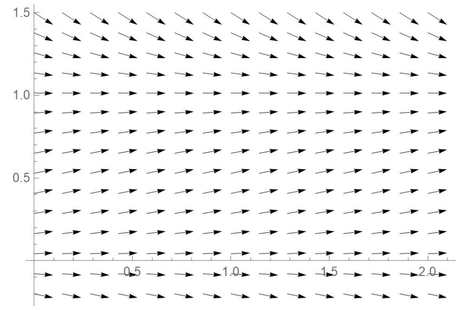
1. Match the direction fields with the differential equations: (use arrows to indicate your matches)



This is $x' = tx(1-x)$



This is $x' = x(t-x)$



This is $x' = x(1-x)$

$\boxed{x' = x(t-x)}$ $\boxed{x' = x(1-x)}$ $\boxed{x' = tx(1-x)}$

2. Solve the initial value problem using separation. Give explicit solution.

$$\frac{dx}{dt} + 6t^2 x = 0, \quad x(0) = -1$$

$\int \frac{dx}{x} = -6\int t^2 dt$ so $\ln|x| = -2t^3 + c$ or $x = Ce^{-2t^3}$.

$\boxed{x(t) = -e^{-2t^3}}$

(turn the page for question 3)

3. Solve $\frac{dx}{dt} + x = t$, $x(0) = 1$

For the standard solution, see version A. Here we show how to "vary a constant", a technique described in Problem 55 on page 48.

The first step is to find the general solution of the homogeneous equation $\frac{dx}{dt} + x = 0$. This is a separable equation so we get $x_h = Ce^{-t}$. Next, we seek a particular solution of the form $x_p(t) = c(t)e^{-t}$ - thus the name of the method - varying a constant.

By product rule, $x_p'(t) = c'(t)e^{-t} - c(t)e^{-t}$. We insert this expression into the left hand side of the equation:

$$x_k' + x = c'(t)e^{-t} - c(t)e^{-t} + c(t)e^{-t} = c'(t)e^{-t}$$

The right hand side of the equation is t so we get $c'(t)e^{-t} = t$ or $c'(t) = te^t$ or $c(t) = \int te^t dt = te^t - e^t$ (here we seek only particular solution so I chose the constant to be 0)

This shows that $x = x_h + x_p = Ce^{-t} + (te^t - e^t)e^{-t} = ce^{-t} + t - 1$. Solving for c from the initial condition, we get $C = 2$, so the answer is $\boxed{x(t) = t - 1 + 2e^{-t}}$