Printed: April 5, 2023 Name ______________________

Dynamical Systems Quiz-10 Key A

Instructions. Be sure to indicate how you got your answers.

1. Consider $\frac{dx}{dt} = x^2 + y^2 - 25$, $\frac{dy}{dt} = 3x - 4y$

   (a) Determine all equilibria and classify (node, saddle, spiral, center, stable or unstable).

   There are two solution of the system

$$\begin{aligned} x^2 + y^2 &= 25, \\ 3x &= 4y : \end{aligned}$$

   $(x, y) = (4, 3)$ and $(x, y) = (-4, -3)$.

   The Jacobi matrix is $J = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x & 2y \\ 3 & -4 \end{bmatrix}$.

   - At $(x, y) = (4, 3)$ $J = \begin{bmatrix} 8 & 6 \\ 3 & -4 \end{bmatrix}$. The eigenvalues are $\lambda_1 = 2 + 3\sqrt{6}, \lambda_2 = 2 - 3\sqrt{6}$ $\boxed{\text{saddle point}}$ with eigenvectors $\vec{v}_1 = \begin{bmatrix} 2 + \sqrt{6} \\ 1 \end{bmatrix} \approx \begin{bmatrix} 4.44949 \\ 1. \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 2 - \sqrt{6} \\ 1 \end{bmatrix} \approx \begin{bmatrix} -0.44949 \\ 1 \end{bmatrix}$
   - At $(x, y) = (-4, -3)$ $J = \begin{bmatrix} -8 & -6 \\ 3 & -4 \end{bmatrix}$. The eigenvalues are $\lambda_1 = -6 + i\sqrt{14}, \lambda_2 = -6 - i\sqrt{14}$. This is $\boxed{\text{stable spiral}}$. To determine the direction of rotation, we compute $\frac{d\vec{x}}{dt}$ near the equilibrium point. At $x = y = -3$ we get $\frac{d\vec{x}}{dt} = \begin{bmatrix} -7 \\ 3 \end{bmatrix}$ so this is $\boxed{\text{stable spiral counter-clockwise}}$

   (b) If an eigenvalue is real, find the eigenvectors.

(c) Graph the phase plane using the phase plane of linearized system.