# Dynamical Systems Quiz-1     A   Key

**Instructions.** Be sure to show your work so that it is clear how you got your answers.

1. Which of the following solves the differential equation $2t\frac{dx}{dt} + x = 0$?

   (a) only $x(t) = t^{1/2}$

   (b) only $x(t) = t^{-1/2}$

   (c) None of the above

   (d) Both of the above

   Explain!   The correct choice is: b.

   Here is how to verify this: if $x(t) = t^{1/2}$ then $x' = \frac{1}{2}t^{-1/2}$ so the left-hand side of the equation is $2tx' + x = 2t\frac{1}{2}t^{-1/2} + t^{1/2} = t^{1/2} + t^{1/2} = 2t^{1/2} \neq 0$

   if $x(t) = t^{-1/2}$ then $x' = -\frac{1}{2}t^{-3/2}$ so the left-hand side of the equation is $2tx' + x = 2t(-\frac{1}{2})t^{-3/2} + t^{-1/2} = -t^{-1/2} + t^{-1/2} = 0$

2. Solve the initial value problem $\frac{dx}{dt} = -2tx^2$, $x(0) = 1$.   $\int \frac{-dx}{x^2} = 2\int t\,dt$ so $\frac{1}{x} = t^2 + c$ and $x = \frac{1}{c+t^2}$. Since $x(0) = 1$, we get $c = 1$ so $x(t) = \frac{1}{1+t^2}$

# Dynamical Systems Quiz-1     B   Key

**Instructions.** Be sure to show your work so that it is clear how you got your answers.

1. Which of the following solves the differential equation $t\frac{dx}{dt} + 2x = 0$?

   (a) only $x(t) = t^{1/2}$

   (b) only $x(t) = t^{-1/2}$

   (c) None of the above

   (d) Both of the above

   Explain!   The correct choice is: c

2. Solve the initial value problem $\frac{dx}{dt} = -6t^2x^2$, $x(0) = 1$.   $\int \frac{-dx}{x^2} = 6\int t^2\,dt$ so $\frac{1}{x} = 2t^3 + c$ and $x = \frac{1}{c+2t^2}$. Since $x(0) = 1$, we get $c = 1$ so $x(t) = \frac{1}{1+2t^3}$

# Dynamical Systems Quiz-1     C   Key

**Instructions.** Be sure to show your work so that it is clear how you got your answers.

1. Which of the following solves the differential equation $2\frac{dx}{dt} - x/t = 0$?

   (a) only $x(t) = t^{1/2}$

   (b) only $x(t) = t^{-1/2}$

   (c) None of the above

   (d) Both of the above

   Explain!   The correct choice is: a

2. Solve the initial value problem $\frac{dx}{dt} = -6tx^2$, $x(0) = 1$.   $\int \frac{-dx}{x^2} = 6\int t\,dt$ so $\frac{1}{x} = 3t^2 + c$ and $x = \frac{1}{c+3t^2}$. Since $x(0) = 1$, we get $c = 1$ so $x(t) = \frac{1}{1+3t^2}$