Printed: April 19, 2021 Name ______________________________

Differential Equations MATH 2073 Final-2019A

Instructions. Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Solve the initial value problem $y' = 2x + y$; $y(0) = 2$.

2. Solve the initial value problem $y' = 6yx$; $y(0) = 6$

3. Find the solution of the equation $y'' - 2y' = 4$ with the initial value $y(0) = 1, y'(0) = 1$.

4. Find the solution of the equation

$$y''' + 4y' = 0 \text{ with initial conditions } y(0) = 1,\ y'(0) = 2,\ y''(0) = 4$$

5. Find the general solution of the equation $y^{(4)} - y = 4e^t$.

6. Find the general solution of the homogeneous Euler equation $x^2y'' - 6y = 0$ for $x > 0$. *Hint. Recall that we search for the solutions of the form* $y = x^r$.

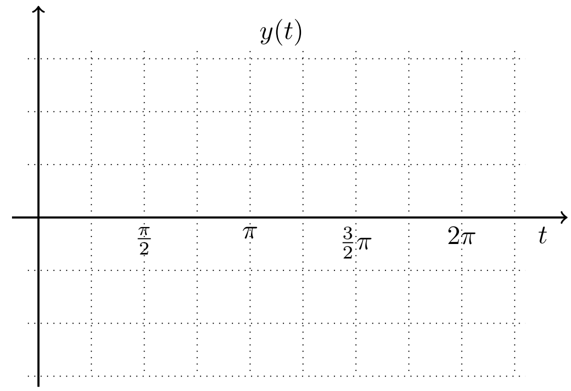
7. Find the power series solution $y = \sum_{n=0}^{\infty} a_n x^n$ of equation $y'' - xy' - y = 0$ with the initial value $y(0) = 1, y'(0) = 0$. Steps in the solution:

(a) Express xy' and y'' as the power series in powers of x^n.

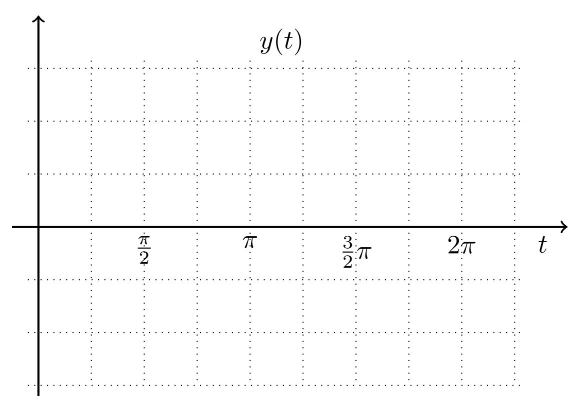
(b) Determine the recurrence relation for coefficients a_n. Simplify your answer!

(c) Compute the first six terms of the power series $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \ldots$ and notice the pattern.

8. Solve the differential equation $y'' + 4y = 4u_\pi(t),\ y(0) = 1, y'(0) = 0$. (Here $u_\pi(t)$ is the unit step function.) Sketch the graph of the solution.



9. Solve the differential equation $y'' + y = \delta(t - \pi), y(0) = 0, y'(0) = 1$. (Here $\delta(t)$ is the unit impulse function.) Sketch the graph of the solution.



10. Consider the system of differential equations

$$\begin{aligned} u_1' &= 2u_1 + u_2, \\ u_2' &= u_1 + 2u_2 \end{aligned}$$

with initial condition $u_1(0) = 4, u_2(0) = 2$.

Determine solution $u_1(t)$ of this system. *Hint. Transform the system into a single equation of order 2. (Alternatively, you can use the Laplace transform.)*