 Name ______________________________

Differential Equations MATH 2073 Final-2018.

Instructions. Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Find the solution of the initial value problem $y' + 6t^2y^2 = 0$, $y(0) = 1$.

2. Solve the initial value problem $y' - 2y = 4t,\ y(0) = 0$.

3. Find the general solution of the equation $y^{(4)} - y = 10e^t$.

4. Find the general solution of the equation $y''' + 4y' = 10e^t$

5. Find the general solution of the nonhomogeneous equation $y'' - 4y' + 3y = 10\cos t$.

6. Find the general solution of the homogeneous Euler equation $x^2y'' + xy' - y = 0$ for $x > 0$. *Hint. Recall that we search for the solutions of the form* $y = x^r$.

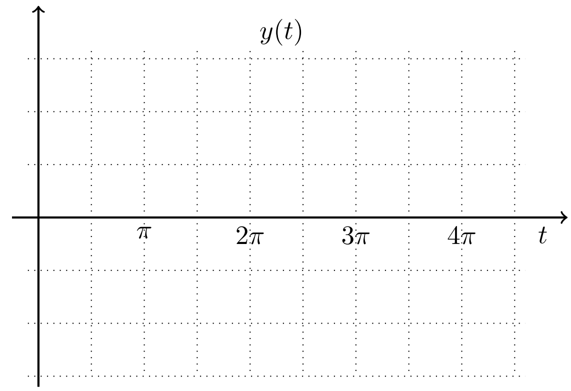
7. Find the power series solution $y = \sum_{n=0}^{\infty} a_n x^n$ of equation $y'' - xy' - y = 0$ with the initial value $y(0) = 1, y'(0) = 0$. Steps in the solution:

(a) Express xy' and y'' as the power series in powers of x^n.

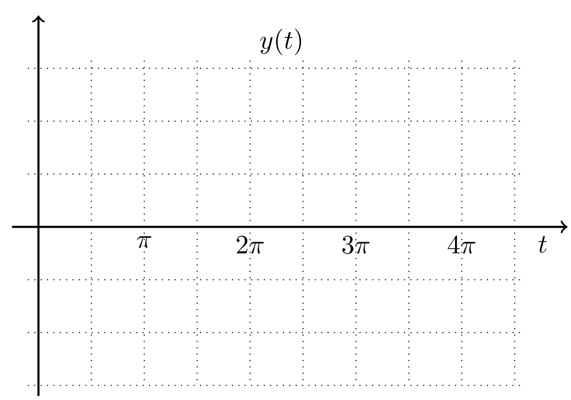
(b) Determine the recurrence relation for coefficients a_n. Simplify your answer!

(c) Compute the first six terms of the power series $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \ldots$ and notice the pattern.

8. Solve the differential equation $y'' + y = -u_\pi(t),\ y(0) = 1, y'(0) = 0$. (Here $u_\pi(t)$ is the unit step function.) Sketch the graph of the solution.



9. Solve the differential equation $y'' + y = \delta(t - \pi), y(0) = 0, y'(0) = 1$. (Here $\delta(t)$ is the unit impulse function.) Sketch the graph of the solution.



10. Consider the system of differential equations

$$\begin{aligned} u_1' &= 2u_1 + u_2, \\ u_2' &= u_1 + 2u_2 \end{aligned}$$

with initial condition $u_1(0) = 2, u_2(0) = 4$.

Determine solution $u_1(t)$ of this system. *Hint. Transform the system into a single equation of order 2. (Alternatively, you can use the Laplace transform.)*

Table of Laplace transforms $\mathcal{L}(f) = \int_0^\infty e^{-ts} f(t) dt$

$f(t)$	$F(s) = \mathcal{L}(f)$	(domain of $F(s)$)
1	$\frac{1}{s}$	$(s > 0)$
t^n	$\frac{n!}{s^{n+1}}$	$(s > 0)$
e^{at}	$\frac{1}{s-a}$	$(s > a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$(s > 0)$
$\sin at$	$\frac{a}{s^2+a^2}$	$(s > 0)$
$u_c(t)$	$\frac{e^{-cs}}{s}$	$(s > 0)$
$\delta(t-a)$	e^{-as}	$(s > 0)$

Laplace transform formulas

$$\mathcal{L}(f_1(t) + cf_2(t)) = \mathcal{L}(f_1(t)) + c\mathcal{L}(f_2(t))$$

$$\mathcal{L}\left(f^{(n)}(t)\right) = s^n F(s) - s^{n-1} f(0) - \cdots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\mathcal{L}\left(e^{at} f(t)\right) = F(s-a)$$

$$\mathcal{L}(t^k f(t)) = (-1)^k \frac{d^k}{ds^k} F(s)$$

$$\mathcal{L}\left(u_c(t) f(t-c)\right) = e^{-cs} F(s)$$

$$\mathcal{L}\left(\int_o^t f(\tau) g(t-\tau)\, d\tau\right) = F(s) \cdot G(s)$$

$$\mathcal{L}\left(f(at)\right) = \frac{1}{a} F\left(\frac{s}{a}\right)$$

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