Decorrelation of the CMB Foreground due to spatially varying spectral index

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1 Introduction

When we look at the greatest cosmic distances, the Universe looks the same everywhere with only tiny departure from homogeneity. This is the Cosmic Microwave Background(CMB), an electromagnetic radiation that fills our sky in every direction. The average temperature of CMB black-body radiation is 2.725 K and is therefore visible in the microwave band of the electromagnetic spectrum. The anisotropy of the CMB consists of small fluctuations in the black-body radiation after Big Bang. According to the standard cosmological model, these are seeded by primordial quantum perturbation that are imprinted onto the early universe immediately after Big Bang by a process called Inflation. Inflation is defined as a period of accelerated expansion within the first fraction of a nanosecond of the universe that explain primordial perturbations as quantum fluctuations in the space-time metric during inflation[0]. The goal of early-universe cosmology is to quantify the observed features of the Universe and to develop a physical model that accounts the observed features.

[0]The quest for B Modes from Inflationary Graviatational Waves

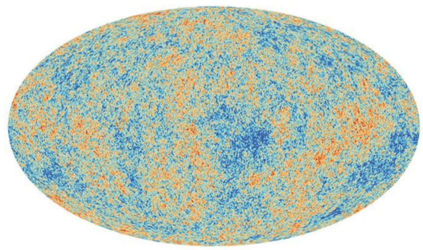

Figure 1: CMB by ESA's Planck Satellite[1]

The *angular power spectrum* of the anisotropy of the CMB contains information about the formation of the universe, it's evolution and it's current contents. The angular power spectrum is defined as a plot of how much the temperature varies from point to point on the sky with respect to the angular frequency, *ell*. However, when observing the CMB which was produced approx 13.8 billions years ago, we are presented with some issues in properly collecting the data in the first place let alone interpret it. Unfortunately, between the true CMB and where we observe it is a long line of obtruding foregrounds which hinder our ability to accurately measure the temperature differences at CMB frequencies. Here we will consider dust maps at four different frequencies ranging from low to high which are: 95,150,220 and 353 GHz. We know that the low and high frequency data are dominated by synchrotron and thermal dust emission respectively[2]. To simulate the dust maps at the high frequency 353 Ghz we use spatially varying spectral index which leads to decorrelation in the power spectrum when scaling to other frequencies. This paper explores:

1. How does the decorrelation vary with the frequency ratio?
2. How does the decorrelation vary with the standard deviation of the $beta_{dust}$ map?
3. How does the decorrelation change if the $beta_{dust}$ maps has different power spectrum i.e $Dl = ell^{\gamma}$

[1] https://www.space.com/33892-cosmic-microwave-background.html

[2]

2 Theory:

2.1 Fourier Transform and Power Spectra

To calculate the angular power spectra of the CMB: We apply the two-dimensional Fourier Transforms (2D FT) to the temperature and polarization(Stokes Q and U) maps after normalization. We estimate D_b^{XY} as a binned version of $D_l^{XY} = \frac{l(l+1)}{2\pi} C_l^{XY}$ where l is the angular frequency called the multipole moment. The reciprocal of the l corresponds to the angular scale which we can call the angular wavelength of the fluctuation. The two indices X and Y can be used to calculate a particular power spectrum of either the {95, 150, 220 or 353} GHz T, E, or B maps.

We make a small angle angle flat sky approximation and take the two dimensional discrete Fourier transform of the product of the maps and apodization mask. The coordinate points in the Fourier space maps are l_x and l_y, which represent modes with wave vector in the direction of right ascension and declination, respectively. The Fourier transformed Q and U maps is then rotated into E, B maps in the Fourier domain as:

$$E(l_x, l_y) = +Q(l_x, l_y)cos2\phi + U(l_x, l_y)sin2\phi$$
$$B(l_x, l_y) = -Q(l_x, l_y)sin2\phi + U(l_x, l_y)cos2\phi$$

Where $\phi = arctan(\frac{l_y}{l_x})$ is the polar angle of each mode with respect to the Fourier plane origin. After transforming to the E and B map the power spectra, D_b^{XY} is calculated as the product of Fourier map X with the complex conjugate of the Fourier map Y. The product is normalized as $l(l+1)/2\pi$, where $l = \sqrt{l_x^2 + l_y^2}$ and then averaged over uniform annular bins.

2.2 Foreground

2.2.1 Dust Simulation:

According to modern precise measurements in cosmology, the foreground dust in our galactic neighbourhood can be modelled as a spectrum that follows power-law on ell. The dust that we observe in the BICEP/KecK maps appears to follow a spectrum with: $Dl_{BB} = A \cdot (ell/ell_{pivot})^{\alpha}$ and $Dl_{EE} = 2 \cdot A \cdot (ell/ell_{pivot})^{\alpha}$[3]. **Note:** the dust has twice as much power in EE than in BB. At 353 GHz: Dust Amplitude(A)=$5\mu K^2$, $ell_{pivot} = 80$ is the angular scale that defines the amplitude, and $\alpha = -0.4$ is the power-law slope.

We can use this known power spectrum to generate map of varying dust spectral index. To create the Stokes Q and U maps of the dust from this power spectrum:

[3] Planck Collaboration, Planck Intermediate Results. XXX. The angular power spectrum of polarized dust emission at intermediate and high Galactic latitudes, Astron. Astrophys. 586, A133, Feb 2016

first calculate Cl using the relation $Cl = \frac{2 \cdot \pi \cdot Dl}{l(l+1)}$. Then calculate the Fourier plane coordinates for size of that map(512×512 in our case): $l = \sqrt{l_x^2 + l_y^2}$ and $\phi = arctan(\frac{l_y}{l_x})$. Next, we create a random one-dimensional piecewise linear interpolation of ell(x-coordinates of the data points) and $\sqrt{Cl}$(y-coordinates of the data points) at l(all points) in the Fourier plane. This gives us the E and B maps in the Fourier plane. Now that we have the E, B maps we can find the Q, U maps in the Fourier plane using the conversion equation from section 2.1, which can then be inverse Fourier transformed back to the real space and normalized using discrete form of parsaval's theorem for Fourier normalization.

2.2.2 Frequency re-scaling:

The dust foreground is caused by the thermal emission from diffuse microscopic particles in our Galaxy heated by by starlight. We would expect thermal emission to obey a blackbody spectrum based on the physical temperature of the dust. However, emission is suppressed at millimeter wavelengths because those wavelengths are physically larger than the typical grain size. This leads to what is known as a "graybody" spectrum, which is a blackbody with emissivity that varies as a power law in frequency, with spectral index β_d(beta_dust) and its intensity, $I_{dust} = \frac{\nu^{3+\beta_d}}{\exp \frac{h\nu}{kT_{dust}} - 1}$. If we have a dust signal with amplitude $1\mu K$ at 353GHz and we want the amplitude at 95GHz, we do the following:

1. Convert $1\mu K$ from thermodynamic units to antenna temperature units at 353GHz.

$$(dP/dT)\,/\,(dP_{RJ}/dT) = \left(\frac{h\nu}{kT_{CMB}}\right)^2 \frac{\exp(h\nu/kT_{CMB})}{(\exp(h\nu/kT_{CMB}) - 1)^2}$$

2. Use power law scaling to go from 353GHz to 95GHz.

$$I_{dust} \propto \frac{\nu^{1+\beta_d}}{\exp(h\nu/kT_{dust}) - 1}$$

3. Convert back from antenna temperature to thermodynamic units at 95GHz.

4. Finally: Scale factor $= \frac{eqn1(@353) \cdot \frac{eqn2(@95)}{eqn2@353}}{eqn1@95}$

2.3 Correlation:

The **correlation amplitude** between two different frequencies for the power spectrum is calculated as follows:

$$95/353\ B - correlation = \frac{Dl_E^{95} \cdot Dl_E^{353}}{\sqrt{Dl_E^{95} \cdot Dl_E^{95} \times Dl_E^{353} \cdot Dl_E^{353}}}$$

The **decorrelation amplitude** is calculated as: $1-$ correlation amplitude(At $ell = 1460$)

3 Data Analysis:

3.1 β_{Dust}: Standard Normal Distribution, $N(1.59, 0.1^2)$

Let us consider the case where the spatially-varying spectral index(β_{dust}) has a Standard Normal Distribution with mean, $\mu = 1.59$ and its standard deviation, $\sigma = 0.1$

3.1.1 Correlation Amplitude as a function of ell

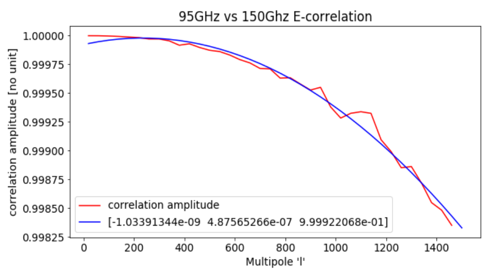

Figure 2: The E-correlation amplitude between 95GHz vs 150GHz seems to be varying quadratically as a function of ell

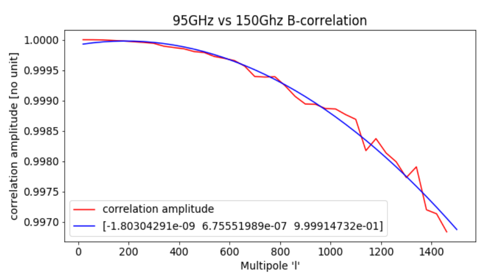

Figure 3: The B-correlation amplitude between 95GHz vs 150GHz seems to be varying quadratically as a function of ell

From figure 2 and figure 3 above, we can see that at the frequency ratio of $0.6333(95GHz vs 150GHz)$ both the E and B correlation amplitude falls off quadratically for increasing ell. However, the B-mode has bigger decorrelation amplitude $= 1 - 0.996832424403127$ compared to the E-Mode decorrelation $= 1 - 0.9983489919598786$.

Figure 4 and Figure 5 shows that the shape of the correlation amplitude remains uncharged for different frequency ratio but it suggest that the decorrelation amplitude increases as the frequency ratio decrease.

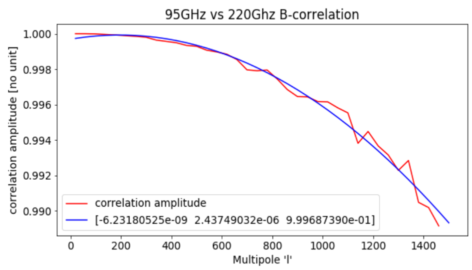

Figure 4: The B-correlation amplitude between 95GHz vs 220GHz seems to be varying quadratically as a function of *ell*

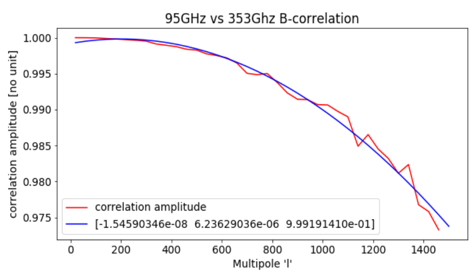

Figure 5: The B-correlation amplitude between 95GHz vs 353GHz seems to be varying quadratically as a function of *ell*

So, for values of frequency ratio=$\{0.2691, 0.4318, 0.6333\}$ we can see that the correlation amplitude always falls off quadritically as *ell* increase. What about the actual value of the decorrelation amplitude? We will check this in the next section.

3.1.2 Decorrelation Amplitude as function of frequency ratio

To see how the actual value of the correlation amplitude depends on the frequency ratio. Let's plot what I defined above in section 2.3 as **decorrelation amplitude** as a function of frequency ratio.

Figure 6 suggest that the decorrelation amplitude falls off quadratically to zero as the frequency ratio approaches 1 which is what we would expect at the same frequency(frequency ratio $\rightarrow 1$), there should be no decorrelation. Note: As mentioned above in section 3.1.1, we can see clearly in this figure that the B-mode decorrelation is almost twice as much as that of the E-mode this is probably because when we simulated the dust map in section 2.2.1, we used

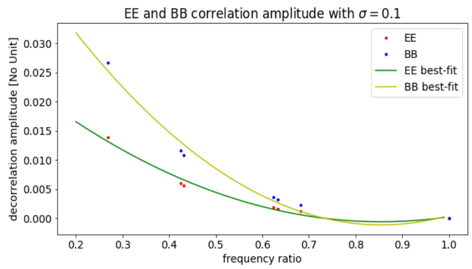

Figure 6: The decorrelation amplitude seems to be quadratic as a function of frequency ratio for $\beta_{dust} = 0.10$

$Dl_{EE} = 2 \cdot Dl_{BB}$ to create the Q and U Fourier map[3]. So, when we rotate the Q, U maps to E, B map using the equation in 2.1 this essentially bleeds twice the spectra of EE into BB giving us a twice the decorrelaion amplitude for the b-mode. Next question we can ask is, how does this change as the standard deviation of the beta_dust varies?

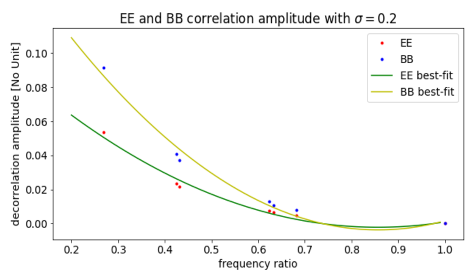

Figure 7: The decorrelation amplitude seems to be quadratic as a function of frequency ratio for $\beta_{dust} = 0.20$

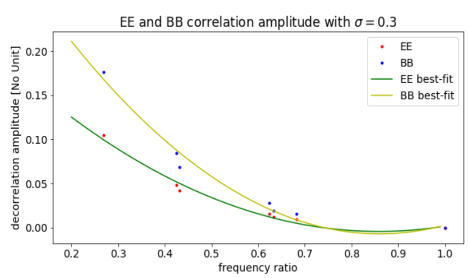

Figure 8: The decorrelation amplitude seems to be quadratic as a function of frequency ratio for $\beta_{dust} = 0.30$

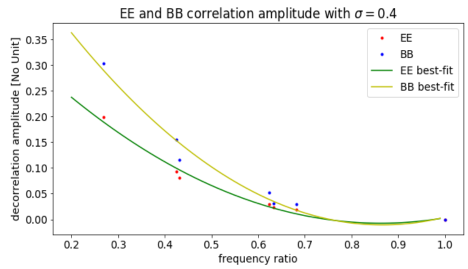

Figure 9: The decorrelation amplitude seems to be quadratic as a function of frequency ratio for $\beta_{dust} = 0.40$

From the plots above with different beta_dust σ. We can deduce that the shape of decorrelation amplitude as a function of frequency ratio remains the same. It is still quadratic however the **decorrelation amplitude increases** as σ of the spatially varying spectral index increases.

3.2 Other Power Spectrum(s) for β_{dust}:

3.2.1 How does the actual dust map change with the change in the power spectrum of the spatially-varying spectral index?

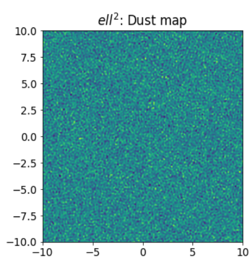

Figure 10: $beta_{dust} : Dl \sim ell^2$

Note: All five of dust maps have been normalized so that they have the same standard deviation, $\sigma = 0.1$.
From the above $beta_{dust}$ maps, we can clearly see that there are some changes to the map. The standard normal dust map(Figure 11) and the $beta_{dust} : Dl \sim ell^2$(Figure 10) are quite similar with smaller feature. The flat dust map(Figure 12) has both small and big features in the map wheres the $beta_{dust} : Dl \sim ell^{-1}$(Figure 13) and $beta_{dust} : Dl \sim ell^{-2}$(Figure 140 have much larger features compared to the standard normal distribution $beta_{dust}$ map.

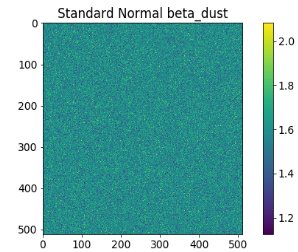

Figure 11: $beta_{dust} : N(1.59, 0.1^2)$

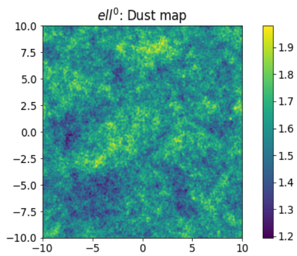

Figure 12: $beta_{dust} : Dl \sim ell^0$

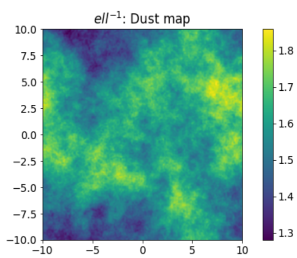

Figure 13: $beta_{dust} : Dl \sim ell^{-1}$

3.2.2 How does shape of the correlation amplitude as a function of ell change with the change in the power spectrum of the spatially-varying spectral index?

let's take a look at the graph of the correlation amplitude as a function of ell at frequency ratio(95vs150) = 0.06333 for all five dust maps in the above section. Figure 15 and figure 16 corresponding to the $beta_{dust} : Dl \sim ell^2$ and $beta_{dust} : N(1.59, 0.1)$ respectively that had similar dust map with smaller features in the section also have similar shape of the correlation function. Their actual values are also quite close and are a pretty good quadratic best-fit. Figure

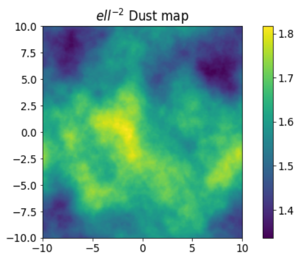

Figure 14: $beta_{dust} : Dl \sim ell^{-2}$

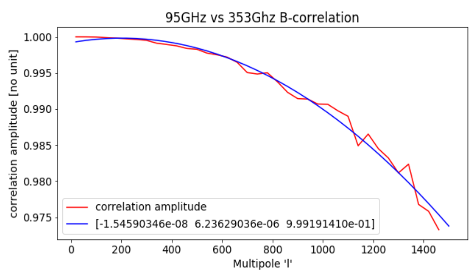

Figure 15: The B-correlation amplitude between 95GHz vs 150GHz with $beta_{dust} : Dl \sim ell^2$

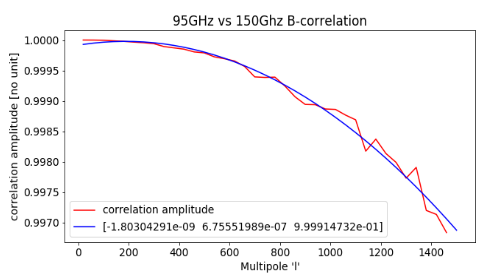

Figure 16: The B-correlation amplitude between 95GHz vs 150GHz with $beta_{dust} : N(1.59, 0.1^2)$

18 and figure 19 corresponding to $beta_{dust} : Dl \sim ell^{-1}$ and $beta_{dust} : Dl \sim ell^{-2}$ respectively that had similar dust maps with larger features also have similar shape of the correlation function. Their actual values are also quite close to each other however is much different from the figure 15 and 16 and its quadratic best-fit is not as good either. Figure 17 that had a mix of both small and big features in the dust map has a correlation function is different from the other four plots and kinds look like a mix of the other two types. It's quadratic best-fit is also not as good as the first two.

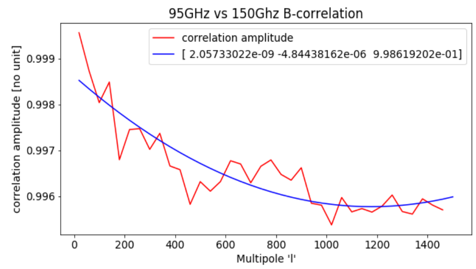

Figure 17: The B-correlation amplitude between 95GHz vs 150GHz with $beta_{dust} : Dl \sim ell^0$

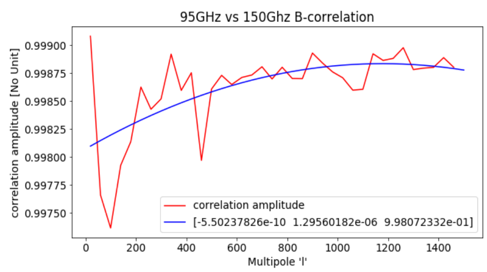

Figure 18: The B-correlation amplitude between 95GHz vs 150GHz with $beta_{dust} : Dl \sim ell^{-1}$

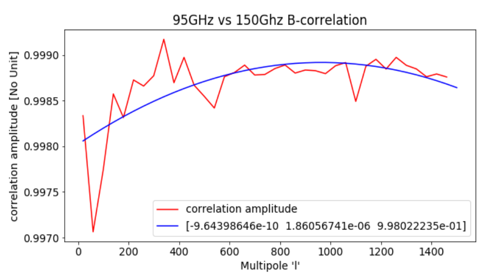

Figure 19: The B-correlation amplitude between 95GHz vs 150GHz with $beta_{dust} : Dl \sim ell^{-2}$

4 Conclusion:

To answer the three questions we proposed at the beginning of the paper. We simulated a dust map at $353GHz$ following the procedure explained in section 2.2.1 then scaled that map to other three frequencies$(95, 150, 220GHZ)$ as described in 2.2.2. Now that we have the dust map at four different frequencies we then calculate their power spectrum as a function of ell, this procedure is

outlined in section 2.1. Next, using the power spectra, we calculate the **correlation** and **decorrelation amplitude** as a function of *ell* as outline in section 2.1 and we are finally ready for data analysis.
From section 3.1.1 we can see that the **correlation amplitude of the power** spectra as a function of *ell* varies quadratically for all frequency ratios. In section 3.1.2 we took a look the decorrelation amplitude as function of frequency ratio. Here also we see that the function is also quadratic and the decorrelation amplitude and approaches 0 and the frequency ration goes to 1 which is what we expect. Although, you can see that the B-mode is more decorrelated than the E-mode in section 3.1.1, this is even more apparent in section 3.1.2. It is because the simulated dust map has twice as much power in EE than in BB. And finally, in section 3.2 we look at and compared different power spectrum for the $beta_{dust}$ maps. Here we find that the correlation does change for different power spectras and that power spectrums that produce similar dust map also create similar decorrelation when frequency scaling.