

Modeling Galactic Foregrounds for Improved CMB B-mode Signal Analysis

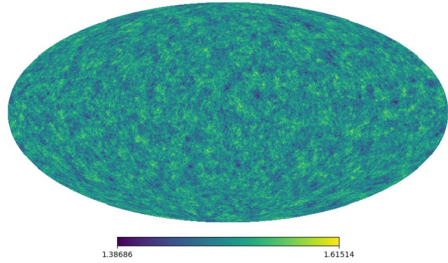
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Why CMB?

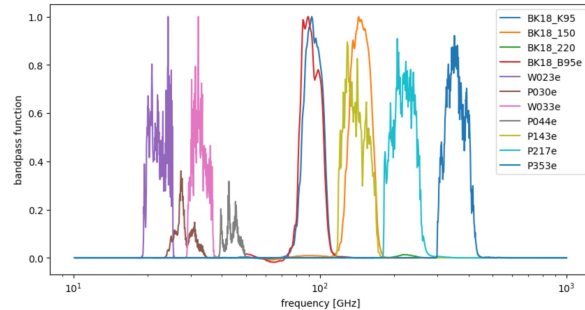
- The Cosmic Microwave Background (CMB) is relic radiation from the early universe, observed ~380,000 years after the Big Bang.
- B-mode polarization in the CMB provides evidence for primordial gravitational waves, supporting cosmic inflation theories.
- Detecting B-modes helps estimate the tensor-to-scalar ratio r , which is crucial for early-universe physics

Foregrounds:

- Major polarized foregrounds: **Synchrotron** (low frequency) and **Thermal Dust Emission** (high frequency).
- These foregrounds obscure faint B-mode signals (~nK level).
- Traditional models assume constant dust parameters (e.g., β_D), but real observations show spatial variability.
- Spatial variation causes **frequency decorrelation**, biasing r .



Simulated Map of Dust Spectral Index β_n demonstrates how dust spectral properties vary across the sky



Instrumental Bandpass Functions centered at different frequencies. This multi-frequency coverage is critical because it allows us to distinguish between the flat spectrum of the CMB and the frequency-dependent foregrounds like thermal dust.

Moments Method – Theory:

Azzoni et al. (2020) introduced a power-spectrum-based formalism that accounts for spatial variations in spectral parameters like β_n . The total cross-power spectrum between two frequencies ν and ν' can be expanded as: $C_l^{\nu\nu'} = C_l^{0x0} + C_l^{1x1} + C_l^{0x2}$. This expansion is then **propagated into the angular power spectrum**, yielding the following terms:

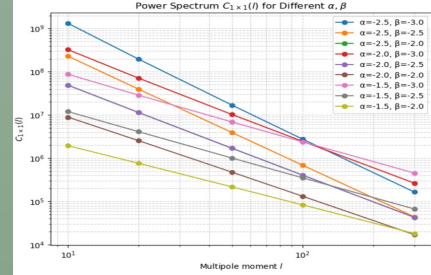
- Baseline spectrum $C_l^{0x0} = S_\nu(\beta_D) \cdot S_{\nu'}(\beta_D) \cdot C_l^{DD}$
- Curvature corrections due to second-order terms $C_l^{0x2} = \sum \frac{1}{2} \cdot (S_\nu(\beta_D) \cdot \partial^2 S_{\nu'}(\beta_D) + \partial^2 S_\nu(\beta_D) \cdot S_{\nu'}(\beta_D)) \cdot C_l^{DD} \sigma^2$
- Correlations of first-order spectral index fluctuations

$$C_l^{1x1} = \sum \partial S_\nu(\beta_D) \cdot \partial S_{\nu'}(\beta_D) \sum_{l_1, l_2} \frac{(2l_1+1)(2l_2+1)}{4\pi} \begin{pmatrix} l & l_1 & l_2 \\ 0 & 0 & 0 \end{pmatrix} C_{l_1}^{DD} \cdot C_{l_2}^\beta$$

Here: S_ν is the average dust spectral energy distribution (SED), modeled as:

$$S_\nu \propto \left(\frac{\nu}{\nu_0}\right)^{\beta_D} \cdot \frac{B_\nu(T_D)}{B_{\nu_0}(T_D)}, \text{ here } B_\nu(T_D) \text{ being the Planck function and } \nu_0 \text{ the pivot frequency}$$

These components track how spectral variability in dust emission impacts the power spectrum at different orders.



Approximated Power-Law Formula for C_l^{1x1} :

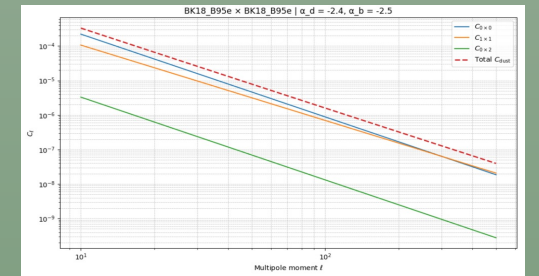
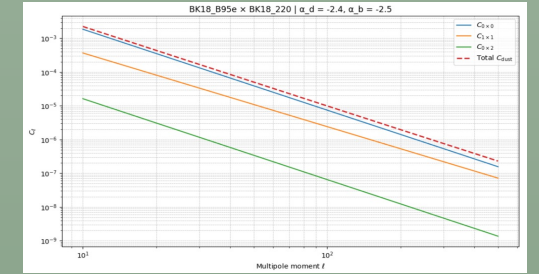
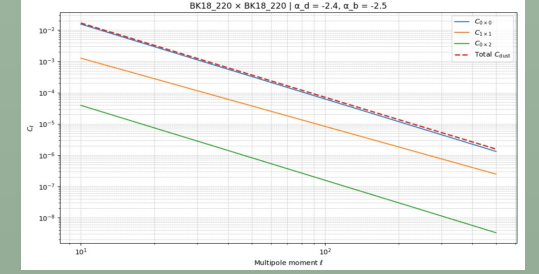
To reduce computation time, we replaced the full Wigner 3j expression with this power-law approximation. The above plot shows the computed values of the moment term C_l^{1x1} , plotted on a log-log scale against l . Each curve exhibits a straight-line trend in log-log space, indicating a clear **power-law behavior**. Thus, making approximations fast enough to use inside MCMC.

$$C_l^{1x1} \approx A_D \left(\frac{l}{l_*}\right)^{\gamma_D}$$

Where we approximated expressions as a function of α_β and α_D , then fitting a regression model using polynomial combinations.

- A_D is the amplitude of the moment term at pivot multipole l^* (80).
- γ_D is the effective slope of the moment term.

Dust Power Spectrum Decomposition Across BICEP/Keck Bandpasses



These plots show how the total dust spectrum is built from the C_l^{0x0} , C_l^{1x1} and C_l^{0x2} terms for a fixed choice of α_β and α_D , and fitted values of A_D and A_β . The 1×1 component speeds up the calculation so that it can be used in a Markov Chain Monte Carlo analysis.

References:

- [1] S Azzoni et al. 2021. Phys Rev D 103(8):083517
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- [3] D Samtleben, S Staggs, and B Winstein. 2007. Annu Rev Nucl Part Sci 57:245–283
- [4] W Hu and M White. 1997. New Astron 2(4):323–344
- [5] BICEP/Keck Collaboration. 2018. Phys Rev Lett 121(22):221301