Blind Separation of CMB from Foregrounds

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Abstract

Attempting to separate the Cosmic Microwave Background (CMB) from complicated foregrounds has always been a challenge. The current methods require assumptions made on the foregrounds. A new method, An Analytical Method of Blind separation of CMB B-Mode from Foregrounds (ABS), to separate the CMB from complicated foregrounds has been proposed by Pengjie Zhang, Jun Zhang, and Le Zhang. This method is attractive for many reasons. The main points are it does not require one to make any assumptions on foregrounds and it does not require a fitting procedure.

1 Introduction

This paper will illustrate the investigation of a new ABS to separate the Cosmic Microwave Background from foregrounds. Separating the CMB from foregrounds has always been a desire to those studying the CMB. The common methods require the researcher to make assumptions on the foregrounds. The attractive aspect of this new method is that there are not any assumptions made on the foregrounds. This paper will be organized into four sections with a conclusion following. In the first section we will understand the motivation behind separating foregrounds from the CMB. Then we will attempt to understand how ABS works. Once we have an understanding of this method we will test the method in various situations. Finally we will see if ABS works for simulated data.

2 Motivation

When measuring the CMB it is impractical to have an uncontaminated CMB signal without several interfering signals. All of these signals lumped together are called foregrounds. For our purposes we will only implement synchrotron and dust radiation into our simple model, and later we will add noise. Instrumental noise also contributes to the observed data in the CMB, and the proposed method has a way of dealing with noise. The blind separation of CMB from foreground's most critical point is that it does not make assumptions based on the foregrounds. This allows us to not worry about what foregrounds are contributing and classify all of them together. Something that is potentially interesting is testing if ABS can also be used to solve for different foregrounds.

3 Theory

Before we dive into the mathematical theory behind ABS we must first understand the data it would be used on. When someone observes the CMB, they collect data at different observing frequencies. For our case, we used 30, 40, 85, 95, 145, 155, 220, and 270 GHz. From these maps, we construct an eight by eight symmetric matrix where the diagonal is map_{30}*map_{30} and the off diagonals are the cross multiplications. These matrices, as mentioned before, have different components. We can view this matrix as:

$$D_{ij}(l) = f_i^B f_j^B D_B(l) + D_{ij}^{fore}(l) \tag{1}$$

We will work in thermodynamic units making $f_j = 1$. Our goal is to solve equation 1 for D_B. We can find an analytical solution due to two facts; D_{ij} has only $M + 1$ eigenvectors and f_j is known for each frequency. Using these facts we can prove that there is the analytical solution:

$$D_B = (\sum_{\mu=1}^{M+1} G_\mu^2 \lambda_\mu^{-1})^{-1} \tag{2}$$

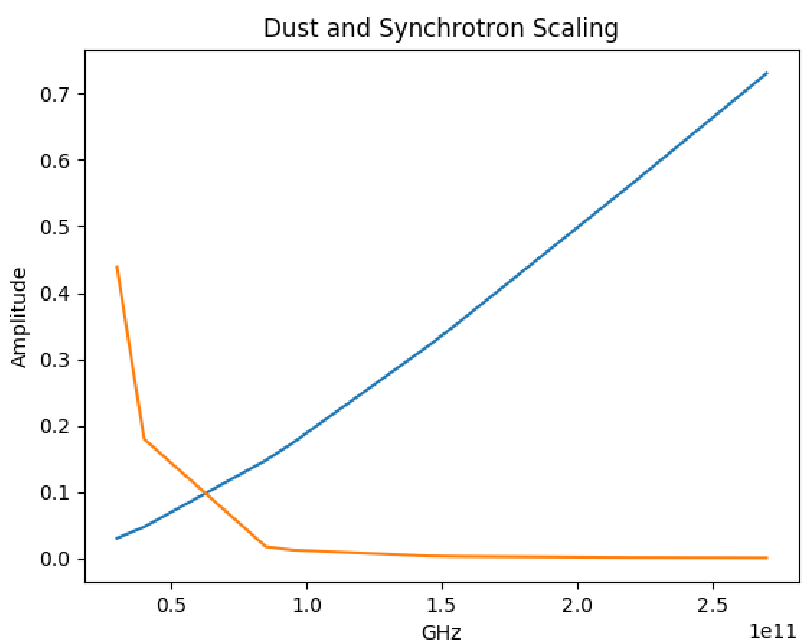
Here $G_\mu \equiv f_B \cdot E^{(\mu)}$ where $E^{(\mu)}$ is the μ-th eigenvector of D_{ij} and λ_μ is the corresponding eigenvalue. Even though this solution may not be straightforward to understand, it is easy to use. Luckily, accounting for noise is easily done by making our cross bandpower matrix into:

$$D_{ij}^{obs} = D_{ij} + \delta D_{ij}^{inst} \tag{3}$$

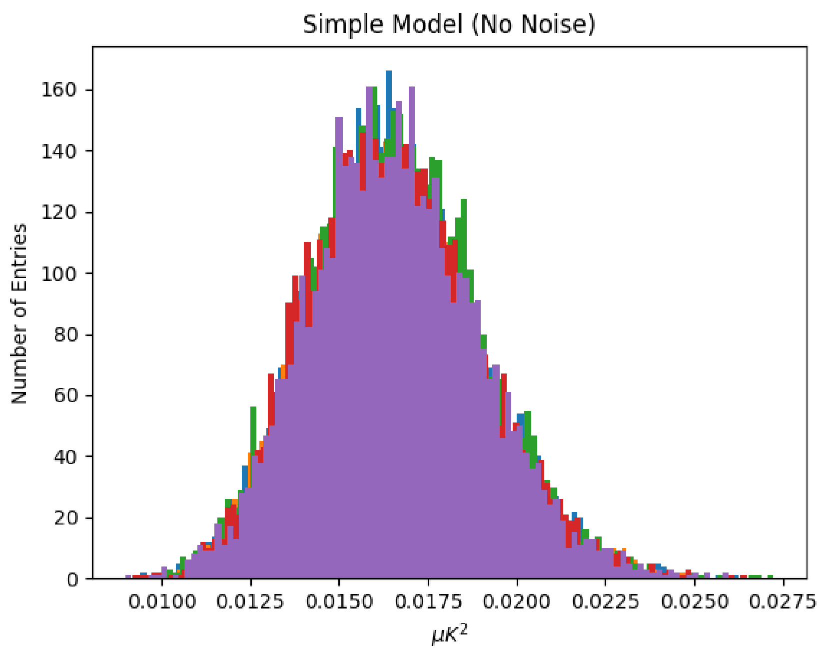
Then to solve for D_B we require $\lambda_\mu > \sigma_D^{inst}$. In later sections We will investigate why and when this can work and still provide the correct answer.

4 Constructing a Model

Our first goal is to construct a simple model. We choose to exclude noise to see if the analytical solution works in the most simple case. To do this we constructed the different components of our cross bandpower matrix. We had three different components: CMB, dust, and synchrotron. For the CMB component, we generated one-hundred random numbers with the same variance, 0.13. This list of random numbers is the same for every frequency. Dust and synchrotron scale differently with frequency. Dust has greater magnitude at higher frequencies than lower. Synchrotron has higher magnitude at lower frequencies than higher. Before proceeding, we checked to make sure we had the scaling of synchrotron and dust radiation correct.



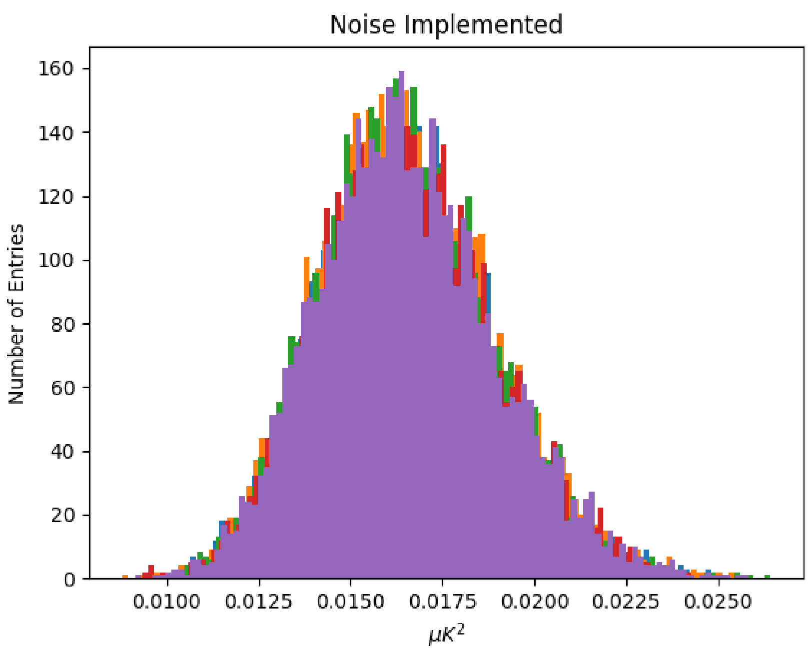
Now we proceed by constructing our dust and synchrotron maps. Again these are a list of one-hundred random numbers with the variance equal to the scaling factors. Using our three maps, CMB, dust, and synchrotron, we can construct our cross bandpower matrix. To get an idea of the accuracy of the algorithm previously discussed, we created 5000 cross bandpower matrices 5 times and plotted them.



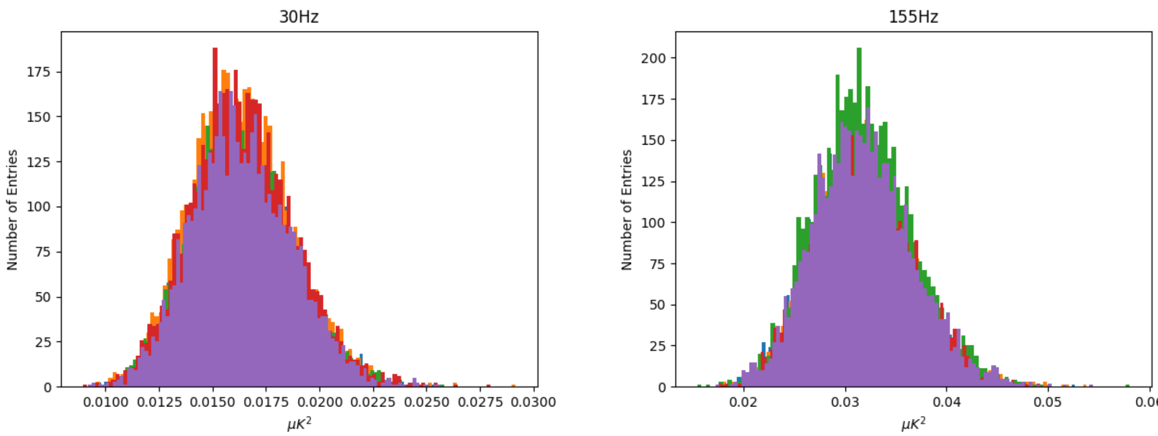
The values center around 0.017, which is our amplitude squared. Here we can conclude that the algorithm works in the most simple cases. To understand the algorithm more, we investigate the eigenvalues for all the eigenvectors. Not to our surprise, there are three eigenvalues that are nonzero. This is to be expected because there are three independent components, dust, synchrotron, and CMB. Next, we investigate if the algorithm can work on more complicated matrices.

4.1 Implementing Noise

First, we construct our noise maps. Again, we make a list of one-hundred random numbers, this time for each frequency. The amplitude depends on the frequency. We used the different noise amplitudes found during calibration of instruments. This lead to the noise amplitudes being at least a couple orders of magnitude lower than the CMB amplitude we chose. Adding our new noise maps as another component in our cross bandpower matrix and adjusting the algorithm to account for noise, we found that there were now eight nonzero eigenvalues. However, five of the eigenvalues were a few orders of magnitude lower than the other three. Naturally, requiring $\lambda\mu > \sigma^D$ only left three eigenvalues, and we see the same problem as before. Doing the same calculations, you can see the plot is very similar:



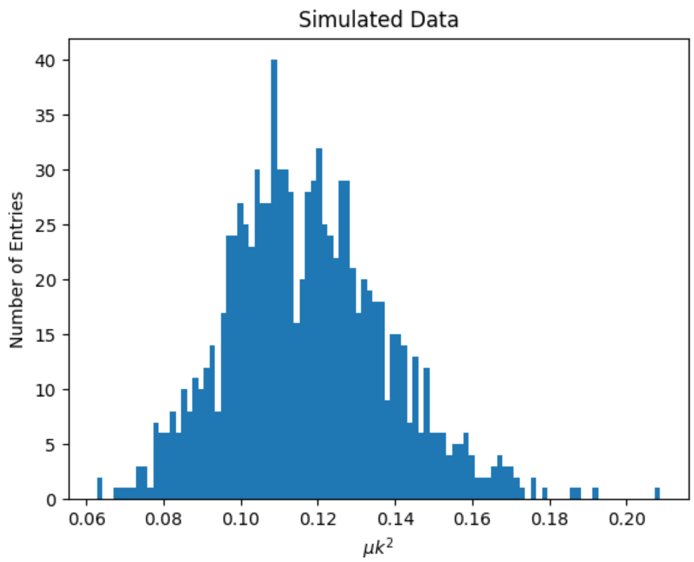
We found that the algorithm works completely in the limit that the instrumental noise is a couple orders of magnitude lower than the CMB signal. Sadly, it is not a given that the noise will be negligible compared to the CMB signal. Testing when and how the algorithm breaks was interesting. Not to our surprise, when the dust amplitudes were close to that of the CMB, the algorithm did not work, nor did it produce anything useful, to my knowledge. To my interest, though, the algorithm depended on some frequencies more than others. For instance, if the noise amplitude at 30Hz was the same order of magnitude as the CMB amplitude, it only skewed the output by a little. However, when the noise amplitude at 155Hz was still an order of 10 smaller than the CMB amplitude, the output was heavily skewed.



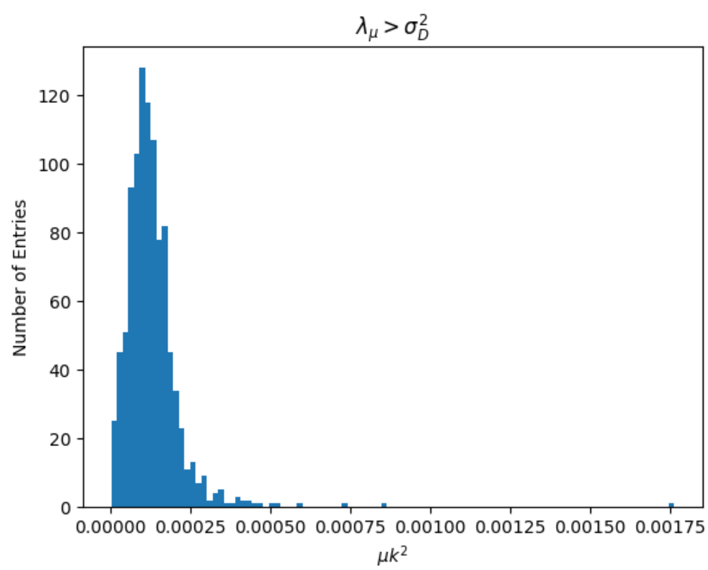
Here you can see the different effects of changing the noise amplitude at 155Hz and 30Hz.

5 Testing ABS on Simulated Data

We used data simulations for the CMB-S4 BICEP/Keck experiment. In these simulations, we had observed data that consisted of a few foregrounds, noise, and CMB. We used this as our D_{ij}^{obs}. There were also simulations of what the noise would be, δD_{ij}^{inst}. If we could get the ABS method to work on the simple foregrounds, there were also simulations of more complicated foregrounds. There was also a file that contained the value of the CMB amplitude. We constructed our matrices our of the data and applied the algorithm to them using the eigenvalues of the noise matrix as the cutoff for the eigenvalues of the D_{ij}^{obs}. At first we used corresponding eigenvalues. For example, if the first eigenvalue of D_{ij}^{obs} was greater than the first eigenvalue of δD_{ij}^{inst} we used the eigenvalue. If it was not, we did not. This result yielded an answer, but it was not corrected according to the expected value.



Here the expected number is $0.003\mu K^2$. We adjusted the cut-off value to see its effect. Being too aggressive with the cut-off value would cause D_B to be zero. Being too passive with the cut-off would cause D_B to be not what we expect.



There was no clear indication that the values we were getting for D_B were related to the answer we were expecting. These results leave us to rethink this algorithm or take a closer look at how the paper takes care of noise.

6 Conclusion

The ABS method seems really attractive in theory. The mathematics behind the method is easily implemented and works efficiently. After some investigation, it appears that we cannot construct good enough instruments to have small enough noise levels. It would be interesting and perhaps beneficial to see if we can obtain any useful information even when the noise levels are too high.