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CMB Polarization Maps & Pure B-Mode Filtering: A Study of the Messenger Method

Abstract

The first component of this project is the development of an understanding of CMB polarization and the intricacies of separating E-mode and B-mode polarization from one another in a partial sky map. The second component of this project is understated in this report but nonetheless vital; the aim is to learn how to use HEALpix to work with maps of the sky and work with real parameters, such as the BICEP/Keck apodization mask. Finally, the third component of this project was to create and apply a Wiener filter to masked maps created from CMB theory spectra. We choose to utilize the Wiener filter because other standard methods of E-B separation involve algorithms that come with too high a computational cost and are cumbersome in terms of switching from pixel space to spherical harmonic space. Here, we use the messenger method to arrive at a Wiener-filtered map that has reduced ambiguous B-modes by a couple orders of magnitude but still hope to better implement the algorithm to further improve the ambiguous B-mode reduction.

Theory/Background

Our story begins fourteen billion years ago, at the moment of the Big Bang. To date, scientists can understand, to an astounding degree of certainty, what happened in the first second of our Universe's life. But the first fraction of a second? That's a different matter. Cosmologists today disagree about what the *very* beginning of the Universe might have looked like, although many agree that the mostly likely scenario is a case of rapid inflation. Anyway, these are questions that no one could even dream of answering until around the 1960s, when researchers finally had the ability to study the cosmic microwave background (CMB) electromagnetic radiation that was caused by recombination as a result of the Big Bang. By observing the CMB intensity pattern, we have been able to learn much about the age, geometry, history, and density of the Universe. To fill in some of those early, missing pieces, we can look for answers in the polarization pattern of the CMB.

We already mentioned that most researchers would agree that inflation is the likely ignition to the start of our Universe. The majority of these researchers would also agree that this inflation happened *rapidly*. In fact, should this be the case, then we would have a definitive explanation as to why the Universe is isotropic, how we came to have galaxies and other structures throughout the sky, and why all indicators suggest that the Universe is flat. The only real problem is that we still do not have concrete evidence to support claims for a rapid explanation, even though we can logically reach such conclusions.

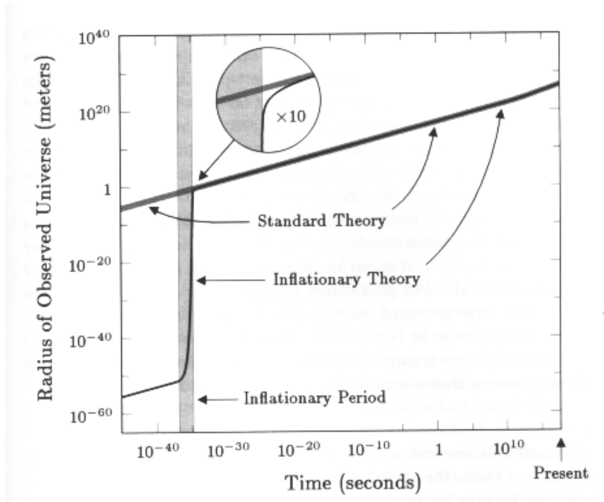


Figure 1 [1]

Figure 1 shows the estimated radius of the Universe in meters as a function of the age of the Universe in seconds. (Note that we are considering a time frame of less than 0.0000000000000000000001 seconds!) This is a visual representation of how quickly a rapidly inflating Universe would expand compared to the linearly expanding Universe that Standard Theory suggests.

Before we look to polarization patterns for our answers, we have to first understand why CMB radiation is polarized to begin with. Consider the abundance of free electrons in space. When those electrons are "hit" by electromagnetic waves caused by the Big Bang, the scattered wave is polarized. This is a phenomenon known as Thomson scattering (see Figure 2). The direction of polarization of the scattered wave is then perpendicular to the direction of the incident wave. Because this radiation is *not* isotropic in nature (in other words, some electromagnetic waves are hotter than others), we see a net polarization. We call this anisotropy of radiation "quadrupole anisotropy" because the poles of anisotropy are separated by ninety degrees.

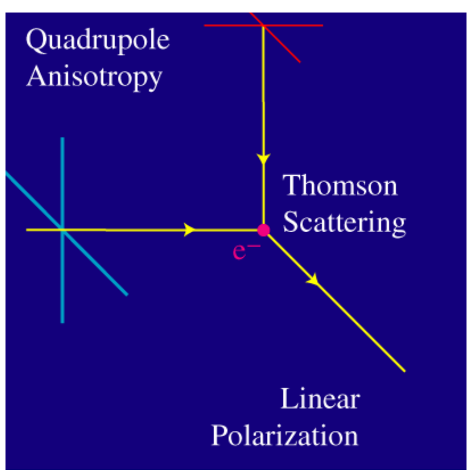


Figure 2 [1]

In Figure 2, the crossed red bars indicate colder radiation, and the blue bars indicate hotter radiation. Each of the electromagnetic waves travels toward the free electron, and then the scattered wave shows the net linear polarization that we then observe in the CMB.

Looking at a polarization map of the CMB, we can see a pattern of polarization that consists of two components, a gradient component and a curl component. The gradient component of the polarization maps corresponds to "E-mode" polarization, named so because of its electric-field like nature. Similarly, "B-mode" polarization corresponds to the curl of the polarization map, termed so because of its similarity to magnetic fields.

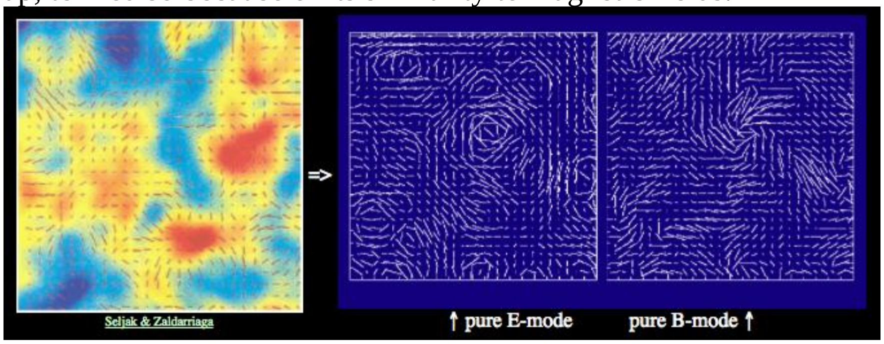


Figure 3 [1]

Notice how the left map in Figure 3 shows a general polarization pattern we might see in the CMB, while the figures on the right show the two components separated gradient and curl components, otherwise pure E-modes, and pure B-modes. (Note that we use the term "pure" to classify these modes because they are *unambiguous* E- and B-modes. Later, we will discuss what we mean when we say modes are ambiguous.)

When looking at these polarization maps, one quickly notices that the magnitude of polarization is quite a bit greater for E-modes than of B-modes, and this is the result of the physical properties that induce the two types of polarization we see. E-modes are so prominent because they are the result of velocity gradients in the plasma. Furthermore, we see peaks in the E-mode spectra at angular scales that correspond to the photon mean free path at the time of decoupling (the average distance traveled by a photon before hitting another photon during recombination). This peak in the spectrum is visible in Figure 4, where we see how the reionization of photons contributes to the polarization spectrum at large angular scales [1]. (Fun fact: the E-mode polarization spectrum and the anisotropy spectrum of CMB temperature are directly out of phase with each other.)

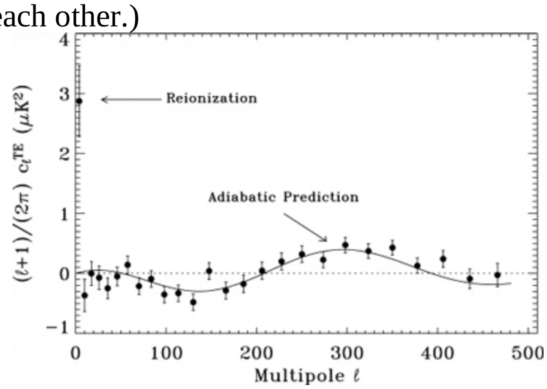


Figure 4 [1]

The B-modes that we have observed to date are actually *secondary* B-modes – polarization caused by gravitational lensing, an observational phenomenon that occurs when we observe a light source so far away that gravity actually warps the light before we detect it. Thus, we see different amplitudes and shapes of polarization because such different physics causes E and B polarization.

In any case, the really exciting topic in CMB research is the search for *primordial* B-modes. Had the Universe begun with rapid expansion, such inflation would have created gravity waves that would cause ripples in the fabric of space. Those gravity waves would then leave their mark as B-mode polarization, much like we see secondary B-mode polarization via gravitational lensing. Thus, detecting primordial B-modes would give us definitive evidence of rapid inflation. The image on the right of Figure 4 shows how we might visualize such a rippling effect.

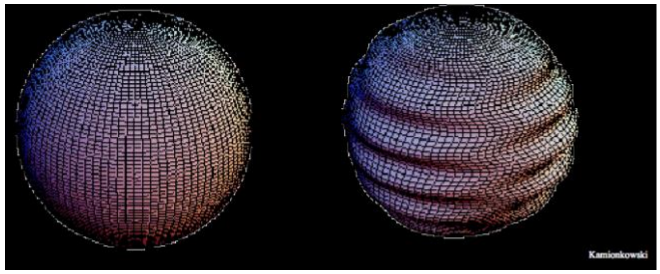


Figure 5 [1]

Now, whenever we consider E and B polarization modes, we use spherical harmonics to decompose and analyze them. This means that whenever we look at actual polarization maps of the CMB, we are not looking at true E and B modes (looking at spherical harmonics in a pixel-spaced map would not make any sense). What we actually see are Stokes parameters Q and U, which describe the linear polarization states of electromagnetic radiation. Q and U represent the direction of polarization. In a polarization map of the CMB, when the direction of polarization matches the direction of wave modulation across the map, we see E-modes. If the direction of polarization is tilted forty-five degrees with respect to the direction of wave modulation, we have detected B-modes.

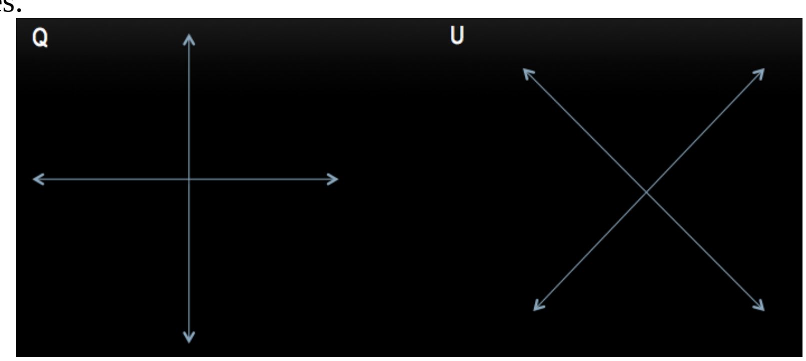
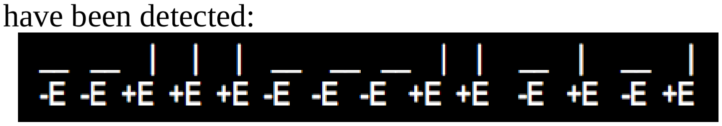


Figure 6

In Figure 6, we denote that Q polarization is vertical and horizontal, while U polarization is “cross” polarization. When we look at our map, if we see Q oscillating horizontally or vertically, E-modes have been detected:



Similarly, if we see U oscillating across the map, we are seeing B-mode polarization:

Even though we are working with concepts in spherical harmonic space, we might also choose to think of how E and B would be analogously related in Fourier space. This is not a perfect metaphor but will give us a way to visualize the physics in a fairly accurate manner.

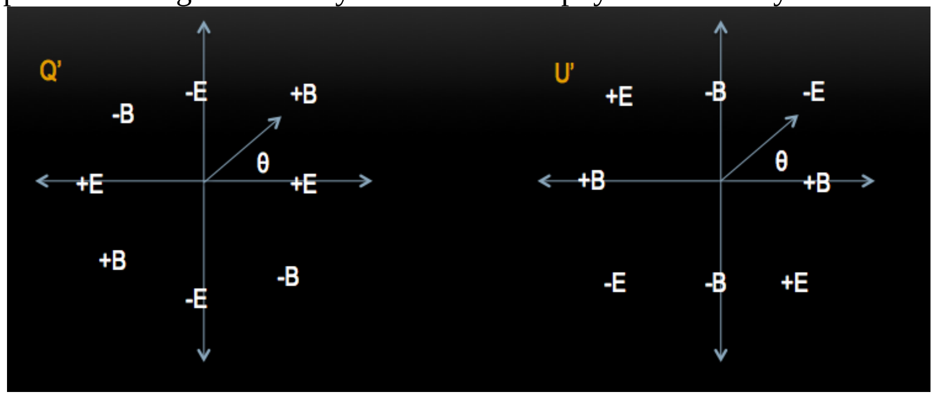


Figure 7

In Figure 7, we denote our Fourier analogy to Q and U as Q' and U' to reiterate that Q' and U' are the Stokes parameters as related to Fourier space and not spherical harmonic space. Anyway, in the left hand plot, we can think of each data point along the x- and y-axis as one E mode that has been detected. Any data point exactly forty-give degrees from the x- or y-axis corresponds to one B mode that has been detected. Any data points in between have both an E-mode and a B-mode component. We can quantify E-modes and B-modes using the following relationship:

$$E' = Q' \cos(2\theta) + U' \sin(2\theta)$$

$$B' = -Q' \sin(2\theta) U' \cos(2\theta)$$

In order to visualize the polarization in CMB maps and develop a sense of how Q and U maps operate, we now take a moment to consider WMAP 5-year data obtained through Harmonic Linear Combination. Note: these are maps created in an attempt to reduce Galactic foregrounds that interfere with the CMB signal.

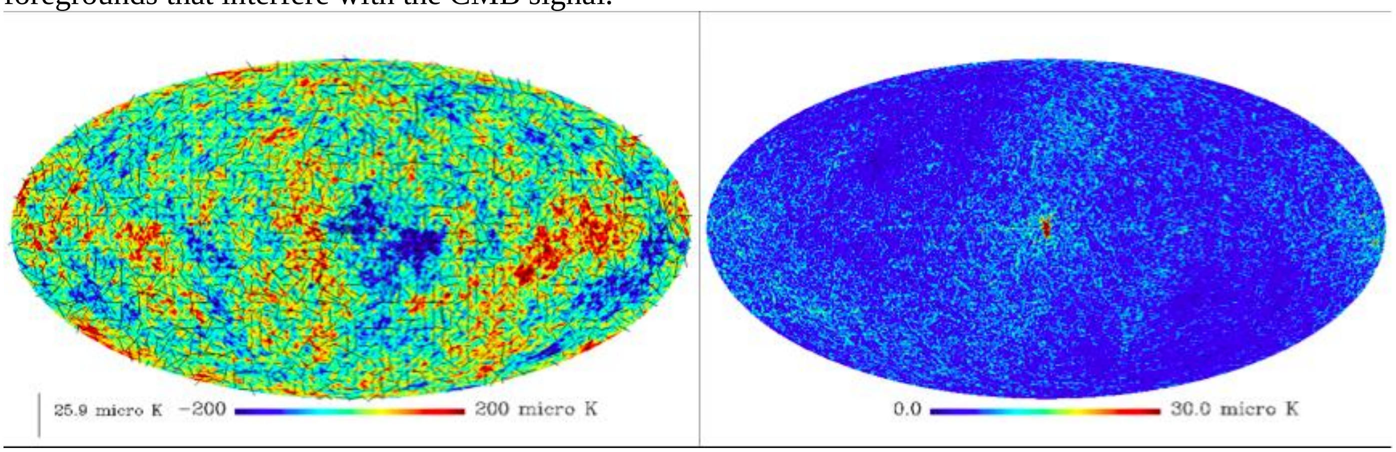


Figure 8 [2]

The image on the left in Figure 8 shows the length of headless polarization vectors proportional to the polarization strength with an underlying color-coded map showing the temperature variance of the CMB. The image on the right shows the magnitude of the polarization vectors. Here, we can also see how the range of temperature magnitude is roughly ten times as large as the range of polarization magnitude. Then, we can also look at maps featuring the Stokes parameter Q and U:

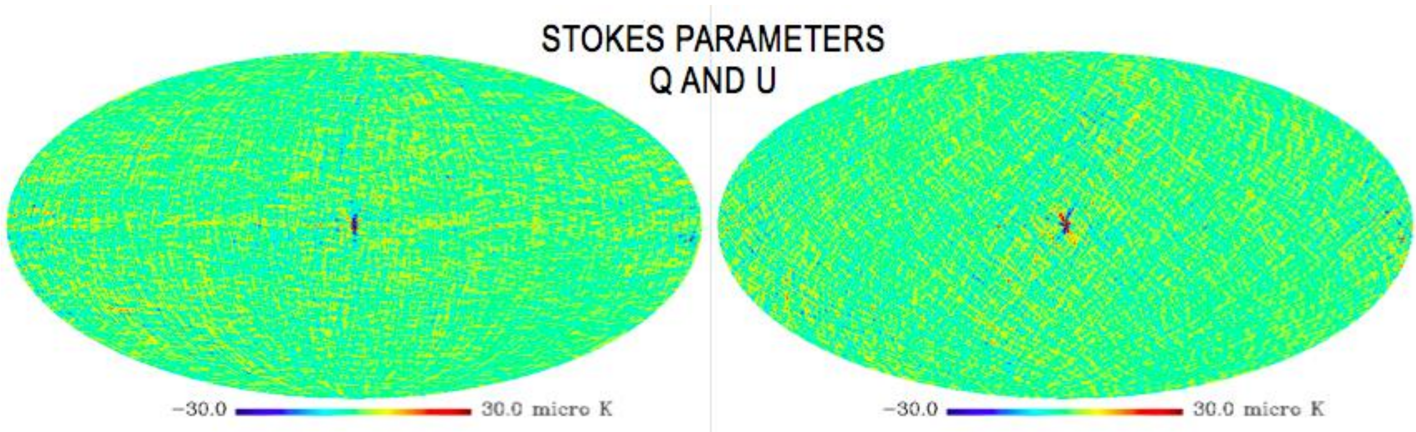


Figure 9 [2]

If we look at the Q map on the left of Figure 9, we see how most of the waves modulate across the map horizontally and vertically (accounting, of course, for the curvature of the map near the edges). As previously accounted, this horizontal and vertical oscillation matches the horizontal and vertical Q polarization, so we have a visual representation of the E-modes we have detected. Likewise, there are also waves in the Q map that modulate diagonally across the map that correspond to B-modes, but they are not as easy to see directly because the magnitude of B-mode polarization is a couple orders of magnitude smaller than the magnitude of polarization of E-modes. Likewise, the figure on the right is a U map, so most of the waves modulate diagonally across the map because, again, they correspond to E-mode polarization.

Creating Masked CMB Polarization Maps

In order to create and use our Wiener filter, we must first create masked polarization maps of the CMB. We begin by importing a set of CMB theory spectra (the derivation of which is beyond the scope of this project), including temperature, E polarization, and B polarization spectra. We set each element of the BB power spectrum to zero to ensure that the only polarization we see in our maps is E-mode polarization. Then, using a program called HEALpix – an algorithm for the pixelisation of a two-dimension sphere, we obtain a set of temperature, Q, and U maps from the spectra, as seen in Figure 10 (note that here we are only concerned with Q and U maps).

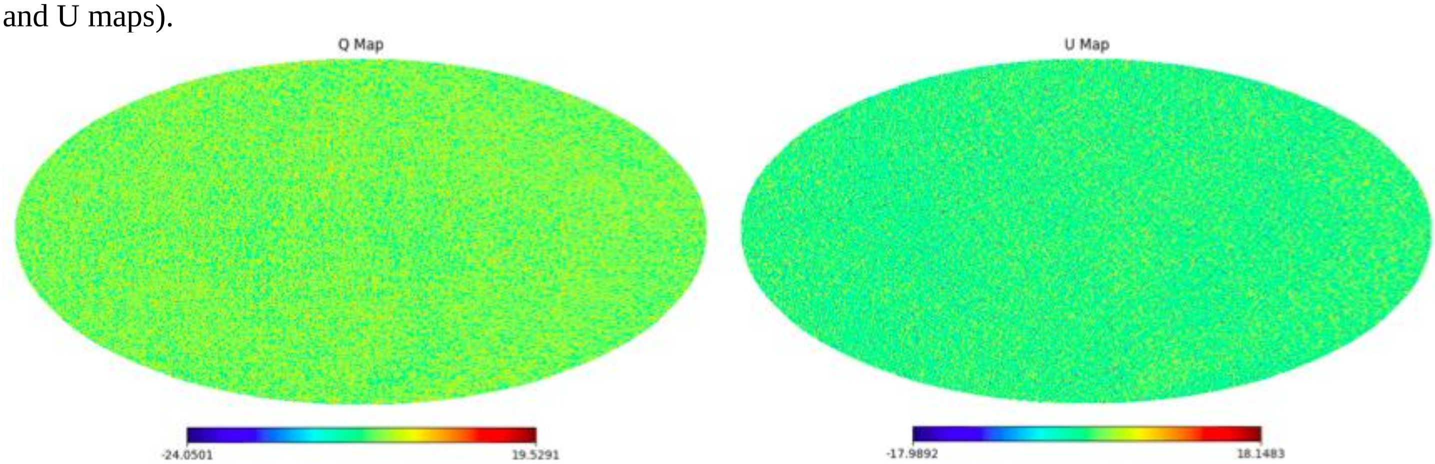


Figure 10

When we have access to full sky maps, such as those in Figures 9 and 10, there is no ambiguity between E and B mode polarization. Each E-mode that is detected is undeniably an E-mode, and each B-mode is likewise a B-mode with no uncertainty. However, obtaining full-sky maps is an expensive and involved process involving orbiting satellites. Often, sky data is obtained via Earth-bound telescopes that are limited to observing some fraction of the sky. Furthermore, full-sky maps are not optimal for many cosmological analyses. Kim, Naselsky, and Christensen tell us that Galactic "foregrounds degrade the attainable accuracy of cosmological information… [and] are particularly significant on large angular scales" [2]. This was the entire reason that their research team worked to reduce these foregrounds via Harmonic Linear Combination, and while they were able to largely reduce foreground contribution, the right-hand image of Figure 8 makes it abundantly clear that a total removal of Galactic foreground has yet to be achieved. Some experiments also are not seeking a shallow image of the CMB, but rather hope to look deeper into the CMB by observing only a very narrow region of the sky. One such example of this is the BICEP and Keck Array CMB Experiments.

To account for the narrow region observed in the BICEP/Keck experiment, we use the apodization mask [3] that corresponds to all of the unobserved regions of the sky and apply it to the maps we created using theory spectra. For every unobserved pixel in the sky, the data value of that pixel is set to zero. In the small region of the sky that the telescope can observe, the data is multiplied by a value from 0 to 1, tapering down as we approach the edge of the mask. Figure 11 shows our Q and U maps once the mask has been applied.

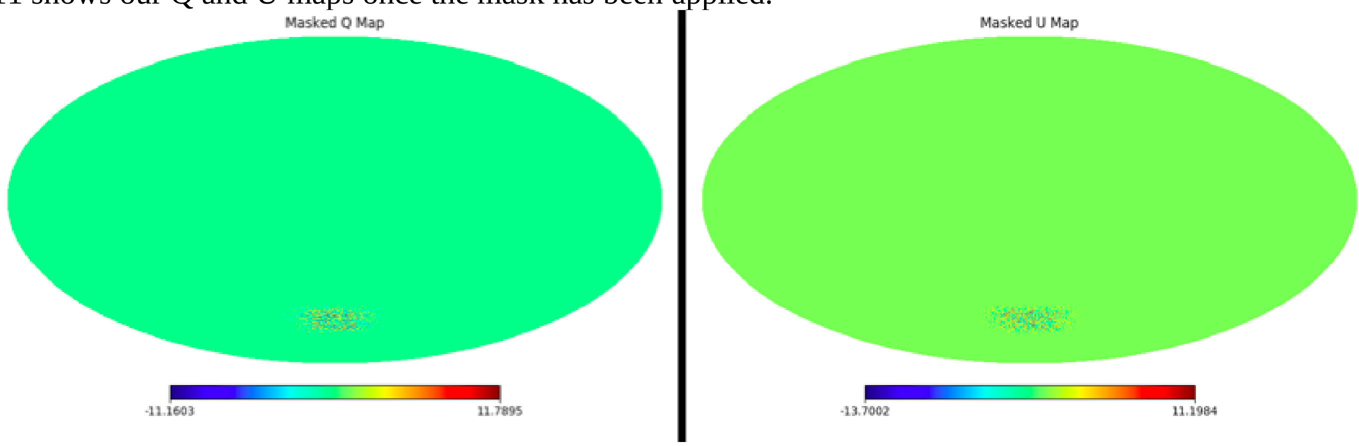


Figure 11

Now that we have our masked maps in pixel space, we can once again use HEALpix to obtain EE and BB power spectra from these maps:

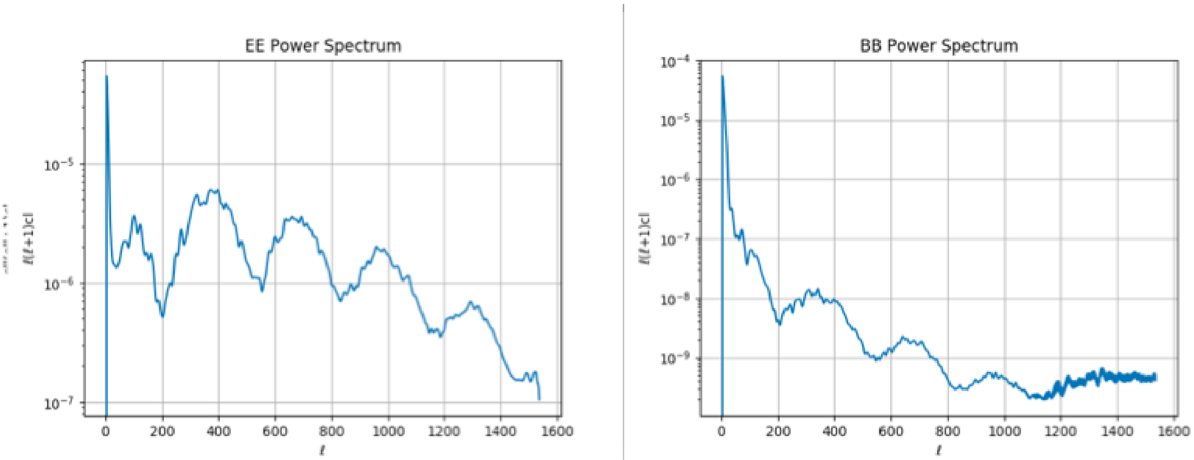


Figure 12

Here the EE power spectrum looks reasonable and corresponds to the E-mode polarization. Recall that when creating our maps initially, we had set every element of the BB power spectrum to zero, resulting in maps that had no B-modes at all. However, upon having masked our maps, we now see that the BB power spectrum is non-zero. In other words, some of the E-modes (primarily around the edge of the mask) have been warped and now appear to be B-modes. This is what we call B-mode leakage and is the result of ambiguous E-modes that are interpreted as B-modes.

Wiener Filtering and the Messenger Method

Our goal is to create a Wiener filter that operates on the original, masked map and leaves only unambiguous B-modes. In the case of the above example, there are no unambiguous B-modes because we removed them before masking the maps. Thus, the Wiener filter that we develop should ideally remove *all* the B-modes from the masked map. If we cannot remove all ambiguous B-modes from the map, we cannot entirely separate E-modes from B-modes, and then we would have no way of detecting primordial B-modes due to the over-powering strength of E-mode polarization.

We begin by establishing a data vector d = s + n, where d is a vector that contains the data from the masked map. Thus, each data vector has a signal component vector, s, and a noise component vector, n. These vectors have corresponding covariance matrices that we designate as **S** and **N**, respectively. We define the Wiener filter map as the vector s_W, which is constructed by minimizing the equation [4]:

$$\vec{s}_W = (\mathbf{S}^{-1} + \mathbf{N}^{-1})^{-1}\mathbf{N}^{-1}\vec{d} = \mathbf{S}(\mathbf{S}+\mathbf{N})^{-1}\vec{d}.$$

Equation 1 [4]

In order to identify the unambiguous B-mode signal, we "lump" the covariance of the E-mode signal, $\mathbf{S_E}$, with the covariance of the noise, **N**. If we think of the pure B-modes as our signal and consider that we are attempting to remove the E-modes from this signal, we accomplish an increase of the signal to noise ratio of our data if we simply treat the E-modes as if they are also noise. This would mean that we can obtain our pure-B Wiener filtered map by replacing the signal covariance matrix **S** in Equation 1 with the B-mode signal covariance matrix $\mathbf{S_B}$ and adding the E-mode signal covariance matrix $\mathbf{S_E}$ to the noise covariance matrix **N**. We define the B and E signal covariance matrices as so in spherical harmonic space [4] :

$$\mathbf{S}_E = \mathrm{diag}(C_2^E, \dots C_{l_{\max}}^E, 0 \dots, 0)$$
$$\mathbf{S}_B = \mathrm{diag}(0, \dots, C_2^B, \dots C_{l_{\max}}^B)$$

Signal Covariance Matrices [4]

Thus Equation 1 becomes [4]:

$$\begin{aligned}\vec{s}_W^B &= (\mathbf{S}_B^{-1} + (\mathbf{S}_E + \mathbf{N})^{-1})^{-1}(\mathbf{S}_E + \mathbf{N})^{-1}\vec{d} \\ &= \mathbf{S}_B(\mathbf{S}_B + (\mathbf{S}_E + \mathbf{N}))^{-1}\vec{d}.\end{aligned}$$

Equation 2 [4]

Our next step is to define a new signal matrix where the E-mode signal is inflated by a factor of α [4]:

$$\mathbf{S}(\alpha) = \mathbf{S}_B + \alpha\mathbf{S}_E$$

Equation 3 [4]

Then Equation 1 becomes:

$$\begin{aligned}\vec{s}_W^B(\alpha) &= \mathbf{S}_B(\mathbf{S}(\alpha) + \mathbf{N})^{-1}\vec{d} \\ &= \mathbf{S}_B\mathbf{S}(\alpha)^{-1}(\mathbf{S}(\alpha)^{-1} + \mathbf{N}^{-1})^{-1}\mathbf{N}^{-1}\vec{d},\end{aligned}$$

Equation 4 [4]

Now if we wish to create a Wiener-filtered map of pure B-modes, we let the E-mode power tend to infinity because as α increases toward infinity, more and more of the ambiguous B-modes are thrown out, in a sense, leaving only the pure B-modes. In this limit, we see a pure-B Wiener-filtered map in the form of [4]:

$$\vec{s}_W^{pB} \equiv \lim_{\alpha\to\infty} \vec{s}_W^B(\alpha) = \mathbf{S}_B\mathbf{S}_B^{+}(\mathbf{S}_B^{+} + \mathbf{N}^{-1})^{-1}\mathbf{N}^{-1}\vec{d}.$$

Equation 5 [4]

One of the primary problems we run into when trying to understand E and B polarization in the CMB is the issue of working in different bases. E and B modes, as previously considered, are defined in the spherical harmonic basis, whereas our maps and our noise exist in map space, or pixel space. In other words, the signal covariance might be sparse in spherical harmonic space, whereas the noise covariance could be sparse in pixel space. The messenger method is an iterative algorithm proposed by Elsner and Wandelt [5] that provides a way to arrive at the solution of the exact Wiener filter equation, written as:

$$\left(\mathbf{S}^{-1} + \mathbf{N}^{-1}\right) s_{\mathrm{WF}} = \mathbf{N}^{-1} d\,.$$

Equation 6 [5]

The set of equations that we use to iteratively solve for the reconstructed signal for s_{WF} are [5]:

$$\left(\bar{\mathbf{N}}^{-1} + (\lambda\mathbf{T})^{-1}\right) t = \bar{\mathbf{N}}^{-1} d + (\lambda\mathbf{T})^{-1} s$$
$$\left(\mathbf{S}^{-1} + (\lambda\mathbf{T})^{-1}\right) s = (\lambda\mathbf{T})^{-1} t\,,$$

Equation 7, Equation 8 [5]

Here, *d* is the data we input from our masked maps, which includes both the data we derive from the theory spectra and the noise associated with the mask. The variable *s* is the signal vector that corresponds to a map that has been filtered of all unambiguous B-modes. **N** is the noise matrix, which in this case corresponds to the apodization mask we use from the BICEP/Keck experiment. **S** is the signal covariance matrix created from the theory spectra. **T** is the covariance matrix of the auxiliary field and we have **T** = τI, where I is the identity matrix and τ = min(diag(**N**)). Therefore, we can think of *t* as a signal reconstruction vector that corresponds to the noise that is evenly distributed in the data. The scalar λ acts as a kind of "cooling" mechanism that has a high initial value but then is reduced after each iteration. As λ approaches 1, we arrive at our pure-B Wiener filtered map [5].

Starting with the first iteration, we begin by initializing the vector *s* with zero. As *s* is our vector of only unambiguous B-modes, we logically must begin by assuming that *none* of the detected B-modes are unambiguous. Then we solve the top equation for *t* and convert *t* from pixel space to spherical harmonic space. Then, we can solve Equation 8 for the vector *s* and convert *s* from spherical harmonic space to pixel space. Now we are ready to begin the second iteration.

If we look once more at Equation 5, we see that going through enough iterations of the messenger method will give us a result for the following term:

$$(\mathbf{S}_B^{+} + \mathbf{N}^{-1})^{-1}\mathbf{N}^{-1}\vec{d}.$$

In order to solve for the true Wiener-filtered pure-B map, we will still need to multiply this by the remaining term:

$$\mathbf{S}_B\mathbf{S}_B^{+}$$

Which is the operator that projects onto the B subspace.

After going through just one iteration of the messenger method with λ = 100, we arrive at the following BB power spectrum:

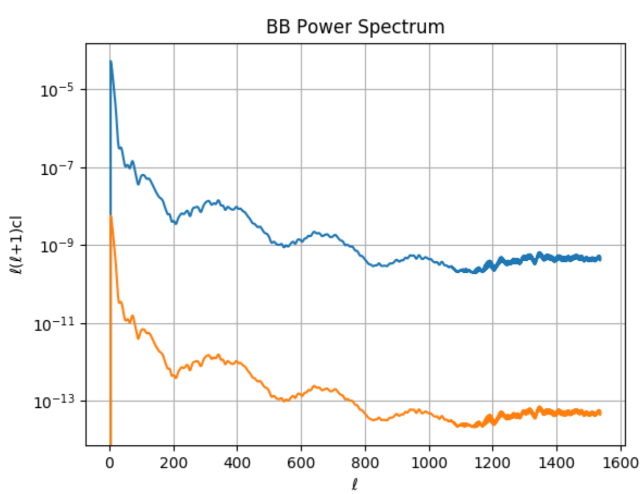


Figure 13

Here, the blue spectrum is the original BB power spectrum prior to any kind of Wiener filter manipulation. The orange spectrum is the new BB power spectrum, and we can see that the B-mode leakage has been reduced by a couple orders of magnitude. Ideally, the B-mode leakage would have been reduced to zero, but as this involved only one iteration of the messenger method, we might still hope to remove more of these ambiguous modes. However, multiplying our original, masked maps by our Wiener filter has indeed gotten rid of some of the B-modes we had hoped to remove.

References

[1] CMB Polarization: The Next Key Toward the Origin of the Universe. Yuki D. Takahashi. http://cosmology.berkeley.edu/~yuki/CMBpol/

[2] CMB polarization map derived from the WMAP 5 year data through Harmonic Internal Linear Combination. Kim, Naselsky, Christensen. https://arxiv.org/pdf/0810.4008.pdf

[3] Keck Array/BISCEP2 October 2015 Data Products. http://www.bicepkeck.org/bk14_2015_release.html

[4] Pure E and B polarization maps via Wiener filtering. Bunn and Wandelt. https://arxiv.org/pdf/1610.03345.pdf

[5] Efficient Wiener filtering without preconditioning. Elsner and Wandelt. https://arxiv.org/pdf/1210.4931.pdf

Other Resources:

MASTER of the CMB Anisotropy Power Spectrum: A Fast Method for Statistical Analysis of Large and Complex CMB Data Sets. Hivon, Górski, Netterfield, Crill, Prunet, and Hansen. https://arxiv.org/pdf/astro-ph/0105302.pdf

Pseudo-C_l estimators which do not mix E and B modes. Smith. https://arxiv.org/pdf/astro-ph/0511629.pdf

Pure pseudo-C_l estimators for CMB B-modes. Smith. https://arxiv.org/pdf/astro-ph/0608662.pdf