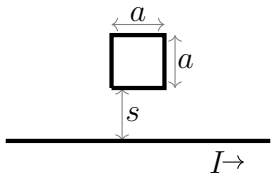
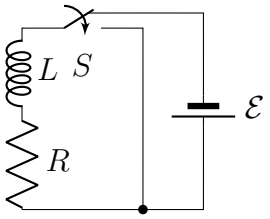
Problem Set 9

Each problem is worth the number of points shown.

Problem 1. (2 points) A square loop, side a, resistance R, lies a distance s from an infinite straight wire that carries current I as shown in the figure below. Starting at time $t = 0$, the current is gradually reduced as $I(t) = (1-\alpha t)I$ for $0 \le t \le 1/\alpha$, and then $I(t) = 0$ for $t > 1/\alpha$. In what direction does the induced current in the square loop flow, and what total charge passes a given point in the loop during the time this current flows?



Problem 2. (2 points) Suppose the circuit in the figure below has been connected for a long time when suddenly, at time $t = 0$, switch S is thrown, bypassing the battery. Find the current as a function of time, $I(t)$, and show that the total energy delivered to the resistor is equal to the energy originally stored in the inductor.



Problem 3. (2 points) A circular cylinder of radius a, carries a constant current, I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in figure 7.43 of the text. Find the electric and magnetic fields in the gap, as functions of the distance s from the axis and the time t, assuming the charge is zero at $t = 0$. Also, find the energy density, $u_{\rm em}$, and the Poynting vector, $\vec{S}$, in the gap. Ignore the fringing fields, i.e., assume $s < a$.

Problem 4. (2 points) Consider an infinite parallel-plate capacitor, whose lower plate at $z = -d/2$ has charge density $-\sigma$, and upper plate at $z = +d/2$ has charge density $+\sigma$. Compute the 9 elements, T_{ij}, of the stress tensor in the region between the plates. What is the momentum per unit area, per unit time, crossing the xy plane?

Problem 5. (2 points) A stationary electric charge q_e has electric field $\vec{E} = (4\pi\epsilon_0)^{-1} q_e \, \boldsymbol{\imath}^{-2} \widehat{\boldsymbol{\imath}}$, and no magnetic field. Similarly, a stationary magnetic monopole q_m has magnetic field $\vec{B} = (4\pi)^{-1} \mu_0 q_m \, \boldsymbol{\imath}^{-2} \widehat{\boldsymbol{\imath}}$, and no electric field. Find the total angular momentum stored in the fields, if the two charges are separated by a distance d.