Implicit Extremes and Implicit Max–Stable Laws

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1. Problem Formulation
2. Limit Theory
3. Implicit Extreme Value Laws
4. Implicit Max–Stable Laws and their Domains of Attraction
5. An Example

Problem Formulation

Setup

- Let $X_1, \ldots, X_n$ be iid vectors in $\mathbb{R}^d$.
- They are hidden, i.e., unobserved.
- Observed are
$$f(X_1), \ldots, f(X_n)$$
for some loss function $f : \mathbb{R}^d \to [0, \infty)$.
- We want to know what is the behavior of the scenario that maximizes the loss
$$X_{k(n)}, \quad \text{where} \quad k(n) = \mathrm{Argmax}_{k=1,\ldots,n} f(X_k).$$
- We refer to $X_{k(n)}$ as to the *implicit extreme* relative to the loss f.

First observations

- If the law of $f(X_i)$ is continuous, with probability one, there are no ties among
$$f(X_1), \ldots, f(X_n)$$
- In the case of ties (discontinuous $\mathcal{L}(f(X_i))$), $k(n)$ is taken as the smallest index maximizing the losses $f(X_i)$, $i = 1, \ldots, n$.
- The motivation stems from applications: We are interested in the structure of the complex (multivariate) events modeled by X_i's that lead to extreme losses.
- These implicit extremes, depending on the loss function f, may or may not be associated with extreme values of the X_i's...
- General Perspective: We are interested in the structure of events leading to extreme losses!

The simple Lemma that started it all

Lemma

Suppose the cdf $G(y) := P(f(X_1) \leq y)$ is continuous. Then, for all measurable $A \subset \mathbb{R}$,

$$P(X_{k(n)} \in A) = n \int_A G(f(x))^{n-1} P_X(\mathrm{d}x)$$

Proof.

There are no ties, a.s., and by symmetry and independence:

$$\begin{aligned} P(X_{k(n)} \in A) &= nP(X_1 \in A,\ f(X_i) \leq f(X_1),\ i = 2, \dots, n) \\ &= n \int_A P(f(X_2) \leq f(x))^{n-1} P_X(\mathrm{d}x). \end{aligned}$$

□

Note: We can handle the general case of discontinuous G.

Limit Theory

Assumptions

- Homogeneous losses: The loss is non-negative $f : \mathbb{R}^d \to [0, \infty)$ and

$$f(cx) = cf(x), \quad \text{for all } c > 0.$$

- This is not a terrible constraint, since

$$\mathrm{Argmax}(f(X_1), \ldots, f(X_n)) = \mathrm{Argmax}(h \circ f(X_1), \ldots, h \circ f(X_n)),$$

for any strictly increasing $h : [0, \infty) \to [-\infty, \infty)$.

- Regular variation on a cone: $P_X \in RV(\{a_n\}, D, \nu)$, where $D \subset \overline{\mathbb{R}}^d$ is a closed cone, playing the role of zero. That is,

$$nP(a_n^{-1}X \in \cdot) \xrightarrow{v} \nu, \quad \text{as } n \to \infty,$$

in the space $\overline{\mathbb{R}}^d_D := \overline{\mathbb{R}}^d \setminus D$.

- This is an important generalization of the usual RV on $\overline{\mathbb{R}}^d_{\{0\}}$.
- Note $RV(\{a_n\}, \{0\}, \nu) \subset RV(\{a_n\}, D, \nu)$. However, the generalized notion of RV allows us to handle cases that are asymptotically trivial in the classical sense.
- Similar (but not the same as) Sid Resnick's hidden regular variation.

RV on cones and generalized polar coordinates

- Consider the compact space $\overline{\mathbb{R}}^d := [-\infty, \infty]^d$.
- Let $\tau : \mathbb{R}^d \to [0, \infty]$ be a continuous and homogeneous function.
- Define $D := \{\tau = 0\}$ (necessarily) a compact in $\mathbb{R}^d$.
- Equip $\overline{\mathbb{R}}_D^d := \overline{\mathbb{R}}^d \setminus D$ with the relative topology.
- The compacts in $\overline{\mathbb{R}}_D^d$ are closed subsets of $\overline{\mathbb{R}}^d$ that are bounded away from $D = \{\tau = 0\}$. That is, $K \subset \overline{\mathbb{R}}_D^d$ is compact if it is closed and $K \subset \{\tau > \epsilon\}$, for some $\epsilon > 0$.
- Polar coordinates: Let $\theta(x) := x/\tau(x)$. Then

$$(\tau, \theta) : \overline{\mathbb{R}}_D^d \to (0, \infty] \times S,$$

is a homeomorphism (of topological spaces), where

$$S = \{\tau = 1\} = \{x \in \overline{\mathbb{R}}^d \ : \ \tau(x) = 1\}$$

is equipped with the relative topology.

Regular variation

Definition

A probability law $P_X \in RV(\{a_n\}, D, \nu)$, if there exists a regularly varying sequence $\{a_n\}$ and a non-trivial Radon measure ν on $\overline{\mathbb{R}}^d_D$, such that

$$nP_X(a_n^{-1}X \in A) \longrightarrow \nu(A), \quad \text{as } n \to \infty,$$

for all measurable A, bounded away from D, i.e., $A \subset \{\tau > \epsilon\}$, for some $\epsilon > 0$, and such that $\nu(\partial A) = 0$.

Fact (Prop 3.8 in Scheffler & Stoev (2014))

$P_X \in RV(\{a_n\}, D, \nu)$, if and only if

$$nP(a_n^{-1}\tau(X) > x) \to_{n\to\infty} Cx^{-\alpha}$$

and

$$P(\theta(X) \in \cdot|\tau(X) > u) \xrightarrow{w}_{u\to\infty} \sigma_0(\cdot),$$

where σ_0 is a finite measure on S.

Example

- **Cone and coors:** Let

$$\tau(x) = \min\{(x_1)_+, \ldots, (x_d)_+\}$$

so that

$$\overline{\mathbb{R}}^d_D = (0, \infty]^d.$$

The unit "sphere", is now:

$$S := \{x : \tau(x) = 1\} = \cup_{i=1}^d [1, \infty]^{i-1} \times \{1\} \times [1, \infty]^{d-i}$$

- **Distribution:** Let $X = (X_i)_{i=1}^d$ with independent and Pareto X_i's

$$P(X_i > x) = x^{-\alpha_i}, \ (\alpha_i > 0), \ i = 1, \ldots, d.$$

Example (cont'd)

The classic RV: In $\mathbb{R}^d_{\{0\}}$, we have asymptotic independence and the heaviest tail dominates:

$$nP(n^{-1/\alpha_*}X \in A) \longrightarrow \mu(A), \quad \text{where } \alpha_* := \min_{i=1,\dots,d} \alpha_i,$$

and

$$\mu(A) = \sum_{i=1}^{d} \mathbb{I}_{\{\alpha_i=\alpha_*\}} \nu_{i,\alpha_*}(A),$$

where $\nu_{i,\alpha}$ is concentrated on the positive part of the i-th axis and

$$\nu_{i,\alpha}(\mathbb{R}^{i-1} \times [x,\infty) \times \mathbb{R}^{d-i}) = x^{-\alpha}, \; x \geq 0.$$

That is, the limit measure μ lives on the axes.

Example (cont'd)

The cone $\mathbb{R}_D^d := (0,\infty]^d$**:** Then, $X \in RV(\{n^{-1/\alpha}\}, D, \nu)$, with

$$\alpha = \alpha_1 + \cdots + \alpha_d$$

and where now ν lives on $(0,\infty)^d$ and now has a density!
Indeed, for $A = (x_1,\infty] \times \cdots \times (x_d,\infty] \subset (0,\infty]^d$,

$$\begin{aligned} P(a_n^{-1}X \in A) &\sim \prod_{i=1}^d P(X_i > a_n x_i) \\ &= \prod_{i=1}^d (a_n x_i)^{-\alpha_i} =: a_n^{-\alpha}\nu(A). \end{aligned}$$

By picking $a_n := n^{-1/\alpha}$, we obtain $nP(a_n^{-1}X \in \cdot) \to^v \nu(\cdot)$, where

$$\frac{\mathrm{d}\nu}{\mathrm{d}x}(x) = \prod_{i=1}^d \alpha_i x_i^{-\alpha_i - 1}, \quad x = (x_i)_{i=1}^d \in (0,\infty]^d.$$

Implicit Extreme Value Laws

Assumptions:

(RV_α) $X \in RV_\alpha(\{a_n\}, D, \nu)$

(H) $f : \overline{\mathbb{R}}^d \to [0, \infty]$ is Borel, 1-homogeneous, $f(0) = 0$.

(F) For all $\epsilon > 0$, the set $\{f > \epsilon\}$ is bounded away from D and

$$\inf_{x \in K} f(x) > 0, \quad \text{for all compact } K \subset \overline{\mathbb{R}}^d_D.$$

(C) $\nu(\overline{\mathrm{disc}}(f)) = 0$.

Theorem (3.13 in Scheffler & Stoev (2014))

Under the above assumptions, we have

$$\frac{1}{a_n} X_{k(n)} \xrightarrow{d} Y, \quad \text{as } n \to \infty,$$

where $P_Y(dx) = e^{-Cf(x)^{-\alpha}} \nu(\mathrm{d}x)$ *and* $C := \nu\{f > 1\}$.

Sketch of the proof

By the above lemma, we have

$$
\begin{aligned}
P(X_{k(n)} \in a_n A) &= n \int_{a_n A} P(f(X) \le f(x))^{n-1} P_X(\mathrm{d}x) \\
&= \int_A P(f(X) \le f(a_n z))^{n-1} n P_{a_n^{-1} X}(\mathrm{d}z) \quad \text{(change of vars)} \\
&= \int_A P(f(a_n^{-1} X) \le f(z))^{n-1} \nu_n(dz) \quad \text{(homogeneity of } f\text{)}
\end{aligned}
$$

where

$$
\nu_n(\mathrm{d}z) := n P_{a_n^{-1} X}(\mathrm{d}z) \equiv n P(a_n^{-1} X \in \mathrm{d}z).
$$

Continuing...

$$
P(X_{k(n)} \in a_n A) = \int_A \left(1 - \frac{n P(f(a_n^{-1} X) > f(z))}{n}\right)^{n-1} \nu_n(\mathrm{d}z).
$$

Sketch of the proof (cont'd)

$$
\begin{aligned}
P(X_{k(n)} \in a_n A) &= \int_A \left(1 - \frac{nP(f(a_n^{-1}X) > f(z))}{n}\right)^{n-1} \nu_n(\mathrm{d}z) \\
&= \int_A \left(1 - \frac{nP(a_n^{-1}X \in \{f > f(z)\})}{n}\right)^{n-1} \nu_n(\mathrm{d}z).
\end{aligned}
$$

The set $B_z := \{f > f(z)\}$ is bounded away from D. If it is a continuity set of ν, by the (RV_α) and (H) assumptions:

$$
\begin{aligned}
nP(a_n^{-1}X \in B_z) &= nP(a_n^{-1}X \in \{f > f(z)\} \longrightarrow \nu(\{f > f(z)\}) \\
&= \nu(f(z) \cdot \{f > 1\}) = f(z)^{-\alpha}\nu\{f > 1\}.
\end{aligned}
$$

Since by (RV_α), we also have $\nu_n \to^v \nu$, it can be shown that

$$
P(X_{k(n)} \in a_n A) \longrightarrow \int_A e^{-\nu\{f>1\}f(z)^{-\alpha}} \nu(\mathrm{d}z),
$$

for all ν-continuity sets A. □

Comments

- The heuristic interchange of 'lim' and '$\int$' in the proof has been justified with some tedious lemmas.
- The measure
$$e^{-\nu\{f>1\}f(z)^{-\alpha}}\nu(\mathrm{d}z) \tag{1}$$
is a probability measure on $\mathbb{R}^d_D$.
That is,
$$\int_{\mathbb{R}^d_D} e^{-\nu\{f>f(z)\}}\nu(\mathrm{d}z) = \int_{\mathbb{R}^d_D} e^{-\nu\{f>1\}f(z)^{-\alpha}}\nu(\mathrm{d}z) = 1.$$
This is amusing and somewhat non–obvious. For example, nothing changes in the limit if $\nu := c\nu$ for $c > 0$, but
$$\int e^{-c\nu\{f>f(z)\}}c\nu(\mathrm{d}z) = 1!$$
- The limit laws in (1) will be referred to as (f,ν)-implicit extreme value laws.

Implicit Extreme Value Laws

Spectral measure

Let $X \in RV_\alpha(\{a_n\}, D, \nu)$. The homogeneity of ν:

$$\nu(cA) = c^{-\alpha}\nu(A), \quad \text{for all } c > 0,$$

implies the disintegration formula

$$\nu(A) = \int_S \int_0^\infty 1_A(\tau\theta) \frac{\alpha \mathrm{d}\tau}{\tau^{\alpha+1}} \sigma(\mathrm{d}\theta),$$

where (τ, θ) are any polar coordinates for $\overline{\mathbb{R}}_D^d$ and

$$\sigma(B) := \nu\{(\tau, \theta) \in [1, \infty] \times B\}, \quad B \subset S.$$

is the spectral measure of ν relative to (τ, θ).

A stochastic representation

Recall the limit (f, ν)-implicit EV law is

$$P_Y(dz) = e^{-Cf(z)^{-\alpha}} \nu(\mathrm{d}z).$$

In polar coordinates $z = \tau\theta$, we have

$$P_Y(\mathrm{d}\tau\sigma(\mathrm{d}\theta)) = e^{-Cf(\tau\theta)^{-\alpha}} \frac{\alpha \mathrm{d}\tau}{\tau^{\alpha+1}} \sigma(\mathrm{d}\theta).$$

This yields the stochastic representation:

Fact (Prop 3.17 in Scheffler & Stoev (2014))

Y is (f, ν)-implicit EV if and only if

$$Y \stackrel{d}{=} Z \frac{\Theta}{g(\Theta)}, \quad \textit{where } g(\theta) = C^{-1/\alpha} f(\theta)$$

(i) Z and Θ are independent
(ii) $P(Z \le x) = e^{-x^{-\alpha}}$, $x > 0$ is standard α-Fréchet
(iii) Θ has distribution $g(\theta)^\alpha \sigma(\mathrm{d}\theta) \propto f(\theta)^\alpha \sigma(\mathrm{d}\theta)$ on the unit sphere S.

Implicit Max–Stability

- For simplicity, let $C = \nu\{f > 1\} = 1$. Then,

$$Y = Z\frac{\Theta}{f(\Theta)}, \quad \text{with } \Theta \sim f(\theta)^\alpha \sigma(\mathrm{d}\theta).$$

- Let $Y_1, \ldots, Y_n$ be independent copies of Y. By homogeneity:

$$f\Big(Z_i \frac{\Theta_i}{f(\Theta_i)}\Big) = \frac{Z_i}{f(\Theta_i)} f(\Theta_i) = Z_i,$$

and hence

$$k(n) = \mathrm{argmax}_{i=1,\ldots,n} f(Y_i) = \mathrm{argmax}_{i=1,\ldots,n} Z_i.$$

- Clearly, by the max-stability of Z

$$Z_{k(n)} = \bigvee_{i=1}^{n} Z_i \stackrel{d}{=} n^{1/\alpha} Z$$

and by the independence of the Z_i's and Θ_i's, we have

$$Y_{k(n)} = Z_{k(n)} \frac{\Theta_{k(n)}}{f(\Theta_{k(n)})} \stackrel{d}{=} n^{1/\alpha} Z \frac{\Theta}{f(\Theta)} = n^{1/\alpha} Y.$$

Implicit Max–Stable Laws: Definition and Characterization.

Definition

A rvec X in $\mathbb{R}^d$ is (strictly) f-implicit max–stable if for all n, exists $a_n > 0$, such that

$$a_n^{-1} X_{k(n)} \stackrel{d}{=} X, \quad \text{with } k(n) = \mathrm{argmax}_{i=1,\dots,n} f(X_i),$$

where X_i's are independent copies of X.

We have shown that for non-negative homogeneous f.

Fact (Theorem 4.2 in Scheffler & Stoev (2014))

The (f,ν)-implicit EV laws are f-implicit max–stable. Conversely, if f is continuous, non–negative and 1-homogeneous, then any f-implicit max–stable law is also an (f,ν)-implicit EV, for some Radon measure ν on $\mathbb{R}^d \setminus \{f=0\}$ such that for some $\alpha > 0$,

$$\nu(cA) = c^{-\alpha}\nu(A), \quad \text{for all } c > 0.$$

Implicit Max–Stable Laws and their DoA

Implicit Max–Stable Laws: Definition and Characterization.

Definition

A rvec X in $\mathbb{R}^d$ is (strictly) f-implicit max–stable if for all n, exists $a_n > 0$, such that

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$$\nu(cA) = c^{-\alpha}\nu(A), \quad \text{for all } c > 0.$$

Characterization of the DoA

Definition

We write $X \in DOA_f(Y)$ for an f-implicit max–stable rvec Y if

$$a_n^{-1} X_{k(n)} \xrightarrow{d} Y, \quad \text{as } n \to \infty. \qquad (2)$$

Fact (Theorem 4.4 in Scheffler & Stoev (2014))

If $f : \overline{\mathbb{R}}^d \to [0, \infty]$ is continuous and 1-homogeneous, then $X \in DOA_f(Y)$ if and only if $X \in RV_\alpha(\{f = 0\}, \nu)$, for some $\alpha > 0$.

Notes:

- Satisfying result – the generalized notion of RV is the right one for implicit Max–DOA!
- The 'if' part is our first implicit limit theorem.
- We will sketch the proof of the 'only if' part.

Characterization of the DoA: Proof of the 'only if' part

Suppose (2) holds, i.e., $a_n^{-1}X_{k(n)} \xrightarrow{d} Y, \; n \to \infty$. Then, by CMT

$$a_n^{-1}f(X_{k(n)}) = a_n^{-1} \max_{i=1,\dots,n} f(X_i) \xrightarrow{d} f(Y).$$

Since $f(X_i)$'s are iid random variables, the classic EVT says:

- $f(Y)$ must be α-Fréchet for some $\alpha > 0$.
- $\{a_n\}$ is RV$(1/\alpha)$ sequence.
- Thus, for some $C > 0$ and all $y > 0$,

$$g_n(y) := P(a_n^{-1}f(X_1) \le y)^{n-1} \to e^{-Cy^{-\alpha}}.$$

- But recall the first Lemma:

$$\begin{aligned} P(a_n^{-1}X_{k(n)} \in A) &= \int_{a_nA} P(a_n^{-1}f(X) \le f(x))^{n-1} nP_X(\mathrm{d}x) \\ &= \int_A g_n(f(z)) nP_{a_n^{-1}X}(\mathrm{d}z) \end{aligned}$$

Cont'd

- Thus, for some $C > 0$ and all $y > 0$,

$$g_n(y) := P(a_n^{-1} f(X_1) \leq y)^{n-1} \to e^{-Cy^{-\alpha}}. \qquad (3)$$

- But recall the first Lemma:

$$\begin{aligned} P(a_n^{-1} X_{k(n)} \in A) &= \int_{a_n A} P(a_n^{-1} f(X) \leq f(x))^{n-1} n P_X(\mathrm{d}x) \\ &= \int_A g_n(f(z)) \underbrace{n P_{a_n^{-1} X}(\mathrm{d}z)}_{=:\nu_n(\mathrm{d}z)} \end{aligned}$$

Goal: Show RV of X, i.e.,

$$\nu_n(\mathrm{d}z) = n P_X(a_n^{-1} X \in \mathrm{d}z) \to^v \nu$$

We have:

- $P(a_n^{-1} X_{k(n)} \in A) = \int_A g_n(f(z)) \nu_n(\mathrm{d}z) \to P_Y(A)$.
- From (3), $g_n(y) \to g(y) := e^{-Cy^{-\alpha}}$.

Finishing the sketch of the proof...

Thus, a type of Radon-Nikodym inversion yields

$$\begin{aligned} \nu_n(A) &= \int_A \frac{1}{g_n(f(z))} P(a_n^{-1} X_{k(n)} \in \mathrm{d}z) \\ &\longrightarrow \int_A \frac{1}{g(f(z))} P_Y(\mathrm{d}z) =: \nu(A), \end{aligned}$$

where in the last relation we used that

- $P(a_n^{-1} X_{k(n)} \in \cdot) = \int_\cdot g_n(f(z)) \mu_n(\mathrm{d}z) \to^w P_Y(\cdot)$.
- From (3), $g_n(y) \to g(y) := e^{-Cy^{-\alpha}}$.

Note: I am glossing over details about justifying the Radon-Nikodym "inversion".

An Example

Pareto–Dirichlet Implicit Max–Stable Laws

- Let $X = (X_i)_{i=1}^d$ where $X_i \sim \mathrm{Pareto}(\alpha_i)$, $i = 1, \dots, d$ are independent.
- Recall $X \in RV_\alpha(D, \nu)$ with $\overline{\mathbb{R}}_D^d = (0, \infty]^d$.
- Consider the 1-homogeneous function

$$f(x) = \left(\frac{1}{x_1} + \cdots + \frac{1}{x_d}\right)$$

Fact (Example 5.1 in Scheffler & Stoev (2014))

The f-implicit max-stable law attracting X is:

$$Y = Z\Theta \equiv \left(\frac{Z}{\xi_1} \;\cdots\; \frac{Z}{\xi_d}\right)^\top,$$

where $Z \sim \alpha-$Fréchet independent of $\xi = \Theta^{-1} \sim \mathrm{Dirichlet}(\alpha_1, \dots, \alpha_d)$.

Why Dirichlet?

WLOG, let $\tau(x) := f(x)$ be the radial and $\theta(x) := x/f(x)$ the angular components of polar coordinates in $(0,\infty]^d$.
Then, by the representation of the (f,ν)-implicit EV laws:

$$Y = Z\frac{\Theta}{f(\Theta)}, \quad \text{where } \Theta \sim f(\theta)^{\alpha}\sigma(\mathrm{d}\theta).$$

Since $f(\theta) = 1$, the distribution of Θ is Uniform w.r.t. the spectral measure σ.
We have

$$\frac{\mathrm{d}\nu}{\mathrm{d}x}(x) \propto x_1^{-\alpha_1 - 1}\cdots x_d^{-\alpha_d - 1}, \quad \alpha = \sum_{i=1}^{d}\alpha_i.$$

and thus

$$\sigma(B) = \nu((f,\theta)\in[1,\infty)\times B) \propto \int_{(f,\theta)\in[1,\infty)\times B}\prod_{i=1}^{d} x_i^{-\alpha_i - 1}dx.$$

By making the change of variables $x_i = f/u_i, \; i = 1, \cdots, d$, where $u_d = 1 - \sum_{i=1}^{d-1} u_i$, we get

$$\mathrm{d}x = \tau^{d-1} \prod_{i=1}^{d-1} u_i^{-2} d\tau \mathrm{d}u_1 \cdots \mathrm{d}u_{d-1}.$$

Which gives

$$\begin{aligned} \sigma(B) &\propto \int_1^\infty \int_{\{u^{-1} \in B\}} \tau^{-\alpha} \prod_{i=1}^{d} u_i^{\alpha_i - 1} \mathrm{d}\tau \mathrm{d}u_1 \cdots \mathrm{d}u_{d-1} \\ &\propto \int_{\{u^{-1} \in B\}} \prod_{i=1}^{d} u_i^{\alpha_i - 1} \mathrm{d}\tau \mathrm{d}u_1 \cdots \mathrm{d}u_{d-1} \\ &\propto P(\xi^{-1} \in B), \end{aligned}$$

for $\xi = (\xi_1 \; \cdots \; \xi_d) \sim \mathrm{Dirichlet}(\alpha_1, \ldots, \alpha_d)$.

Thank you!


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