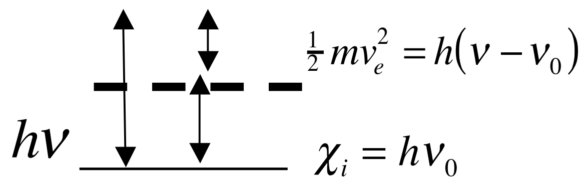
9 - THERMAL EQUILIBRIUM IN THE GAS

Heating ⇒ Photoionization

Cooling ⇒ Recombination
⇒ Bremsstrahlung (“free-free”)
⇒ Collisional

Gains versus Losses in Steady-State: $G_P = L_R + L_{FF} + L_C$

$\boxed{G_P}$



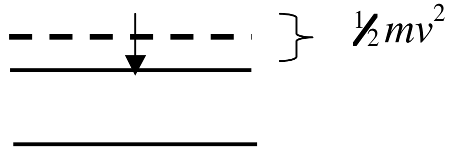
It is possible to show that $\underbrace{G}_{\substack{\text{energy input}\\ \text{rate per vol}}} = \underbrace{n_e n_p \alpha_A}_{\substack{\text{recomb}\\ \text{rate per vol}}} \cdot \underbrace{\frac{3}{2} k T_i}_{\substack{\text{energy input per recomb}\\ \text{(actually per ionization)}}}$ (Note: use α_B to correct for diffuse ionizing radiation).

For stars with blackbody-like spectra, $T_i \approx \frac{2}{3} T_*$

TABLE 3.1
Mean input energy of photoelectrons

Model stellar atmosphere T_*(° K)	T_i(° K)			
	$\tau_0 = 0$	$\tau_0 = 1$	$\tau_0 = 5$	$\tau_0 = 10$
3.0×10^4	1.46×10^4	1.81×10^4	3.61×10^4	5.45×10^4
3.5×10^4	2.15×10^4	2.65×10^4	4.67×10^4	6.31×10^4
4.0×10^4	2.67×10^4	3.38×10^4	6.52×10^4	9.57×10^4
5.0×10^4	3.50×10^4	4.47×10^4	8.47×10^4	11.87×10^4

$\boxed{L_R}$



$$L_R = n_+ n_e \sum_n \sum_L \int_0^\infty v\sigma_{nL}\left(\tfrac{1}{2}mv^2\right) f(v)\,dv$$

$$= n_+ n_e \sum_n \beta_n kT$$

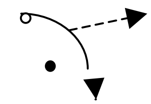
$$= n_+ n_e \beta_B kT \qquad \beta_B = \sum_{n=2}^{\infty} \beta_n \quad (\textit{correct for diffuse field})$$

TABLE 3.2
Recombination cooling coefficient[a]

$T(°K)$	β_A	β_1	β_B
2,500	8.93×10^{-13}	3.13×10^{-13}	5.80×10^{-13}
5,000	5.42×10^{-13}	2.20×10^{-13}	3.22×10^{-13}
10,000	3.23×10^{-13}	1.50×10^{-13}	1.73×10^{-13}
20,000	1.88×10^{-13}	9.58×10^{-14}	9.17×10^{-14}

[a] In $cm^3\ sec^{-1}$.

$\boxed{L_{FF}}$

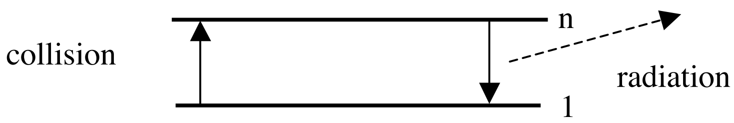


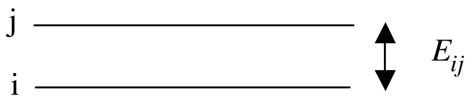
$$L_{FF} = \frac{2^5 \pi e Z^2}{3^{3/2} hmc^3}\left(\frac{2\pi kT}{m}\right)^{1/2} g_{ff} n_e n_+$$

$$= 1.42x10^{-27} Z^2 T^{1/2} g_{ff} n_e n_+$$

$$\sim L_R/3$$

$\boxed{L_C}$ THE MOST IMPORTANT COOLANT! O^+, O^{++}, N^+, etc.





$$LOSS\ RATE\ \#cm^{-3}s^{-1} = \frac{L_C}{E_{ij}} = n_a n_e \int_v^\infty v\sigma(v) f(v) dv \quad where\ \tfrac{1}{2} mv^2 = E_{ij}$$

$$\sigma_{ij} = \frac{\pi\hbar^2}{m^2 v^2}\frac{\Omega_{ij}}{\omega_i}$$

Let:

$\Omega_{ij} =$ "*collision strength*"

$\omega_i =$ *statistical weight of lower level*

$\frac{\hbar}{mv} \sim$ *de Broglie wavelength* λ (*radius*) *of collider*

$$\frac{L_C}{E_{ij}} = n_a n_e q_{ij} \quad or \quad L_C = n_a n_e q_{ij} E_{ij} = n_a n_e q_{ij} h\nu_{ij} \quad where$$

After substituting for σ and f(v), get

$$q_{ij} = \frac{8.63x10^{-6}}{T^{1/2}} \frac{\Omega_{ij}}{\omega_i} e^{-Eij/kT}$$

$$Similarly, \quad q_{ji} = \frac{8.63x10^{-6}}{T^{1/2}} \frac{\Omega_{ji}}{\omega_j} \quad (no\ energy\ threshold)$$

$$ALSO, \quad \Omega_{ij} = \Omega_{ji} \sim 1$$

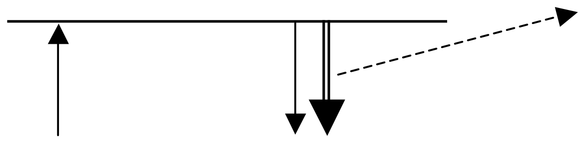
“FORBIDDEN TRANSITIONS” have A_{21}~1 s^{-1} instead of 10^8 s^{-1}. The species giving rise to these transitions are enclosed in braces – “[ ]” is “forbidden”, while “]” is “semiforbidden”.

2 LEVEL ATOM, LOW DENSITY LIMIT (every collisional excitation results in a radiative deexcitation transition – no downward collisions):

$$L_C = n_e n_1 q_{12} E_{12}$$

[Note: downward collisions in the low density regimes tend to have t^{-1}~10^{-3} s^{-1} – even slower than forbidden radiative ones!]

Generally, $n_1 n_e q_{12} = n_2\left(n_e q_{21} + A_{21}\right)$



$$\frac{n_2}{n_1} = \frac{n_e q_{12}}{A_{21}}\left[\frac{1}{1+\frac{n_e q_{21}}{A_{21}}}\right]$$

And $L_C = n_2 A_{21} E_{21} = n_1 n_e q_{12} \frac{E_{21}}{1+\frac{n_e q_{21}}{A_{21}}}$

(only radiative transitions down contribute to cooling)

n→0 get $L_C = n_e n_1 q_{12} E_{12}$ as before.

n→∞ get $L_C = n_1 \frac{\omega_2}{\omega_1} e^{-E/kT} A_{21} E_{21}$ which is the Boltzmann rate, and the system is essentially in thermodynamic equilibrium.

THE DOWNWARD COLLISIONAL AND RADIATIVE TRANSITIONS WILL BE EQUAL AT THE "CRITICAL DENSITY"

$$n_e = \frac{A_{21}}{q_{21}}$$

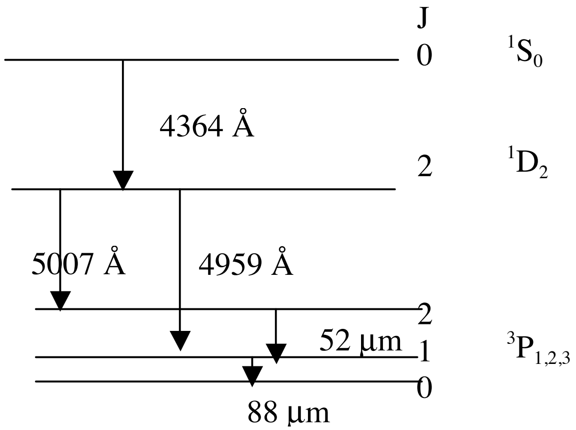
($^{2S+1}L_J$ notation)

TABLE 3.11
Critical densities for collisional de-excitation

Ion	Level	$N_c(\mathrm{cm}^{-3})$	Ion	Level	$N_c(\mathrm{cm}^{-3})$
C II	$^2P_{3/2}$	8.5×10^1	O III	$^1D_2$	7.0×10^5
			O III	$^3P_2$	3.8×10^3
C III	$^3P_2$	5.4×10^5	O III	$^3P_1$	1.7×10^3
N II	$^1D_2$	8.6×10^4	Ne II	$^2P_{1/2}$	6.6×10^5
N II	$^3P_2$	3.1×10^2			
N II	$^3P_1$	1.8×10^2	Ne III	$^1D_2$	7.9×10^6
			Ne III	$^3P_0$	2.0×10^4
N III	$^2P_{3/2}$	3.2×10^3	Ne III	$^3P_1$	1.8×10^5
N IV	$^3P_2$	1.4×10^6	Ne V	$^1D_2$	1.6×10^7
			Ne V	$^3P_2$	3.8×10^5
O II	$^2D_{3/2}$	1.6×10^4	Ne V	$^3P_1$	1.8×10^5
O II	$^2D_{5/2}$	3.1×10^3			

NOTE: All values are calculated for $T = 10,000°$ K.

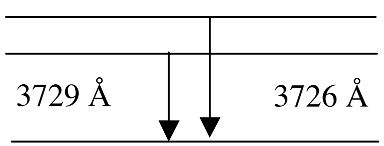
Example – O^{++} (i.e. [O III]) $^{2S+1}L_J$ notation, λ in Ångstroms (10^{-8} cm) or μm



A_{5007}=0.020 s^{-1}
A_{4959}=0.007 s^{-1}

λ5007 is one of the main coolants in H II regions 52 & 88 μm also important.

Example – O^{+}



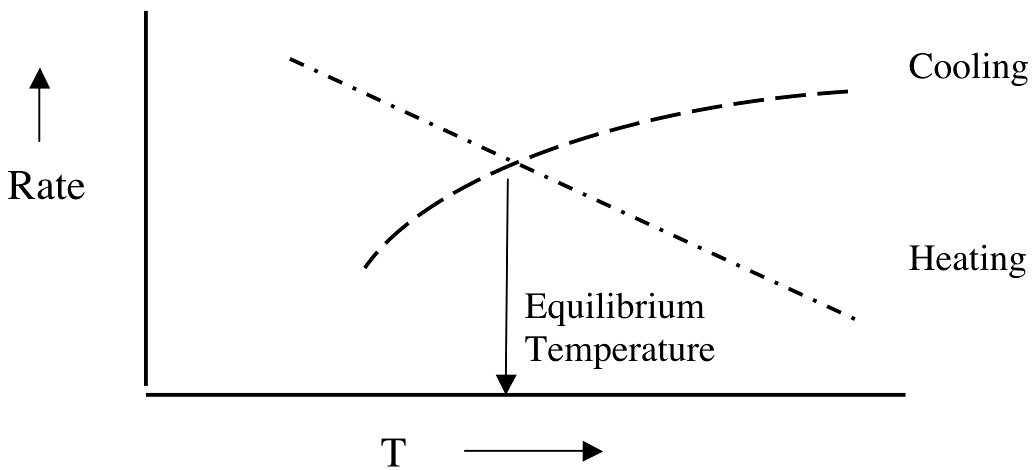
MAJOR BRIGHT LINES

[O III]	λ5007, 4959, 52μm, 88μm
[N II]	λ6583, 6548
[Ne II]	λ12.8μm
[O II]	3726, 3729
H	λ6563, 4861, etc.
[C II]	λ158μm

THERMAL EQUILIBRIUM RESULTS

$$G = L_R + L_{FF} + L_C$$

$$\underbrace{G - L_R}_{\substack{efffective\ \ heating \\ photion.-recomb\,.}} = L_{FF} + L_C$$



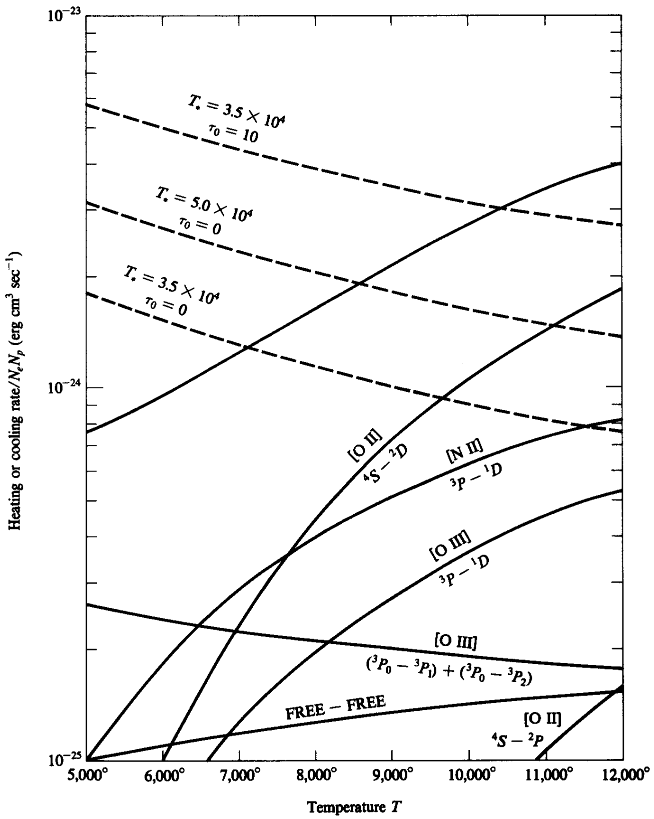


FIGURE 3.2

Net effective heating rates (G-L_R) for various stellar input spectra, shown as dashed curves. Total radiative cooling rate ($L_{FF} + L_C$) for the simple approximation to the H II region described in the text is shown as highest solid black curve, and the most important individual contributors to radiative cooling are shown by labeled solid curves. The equilibrium temperature is given by the intersection of a dashed curve and the highest solid curve. Note how the increased optical depth τ_0 or increased stellar temperature T_* increases T by increasing G.

Low density limit – few collisional deexcitations

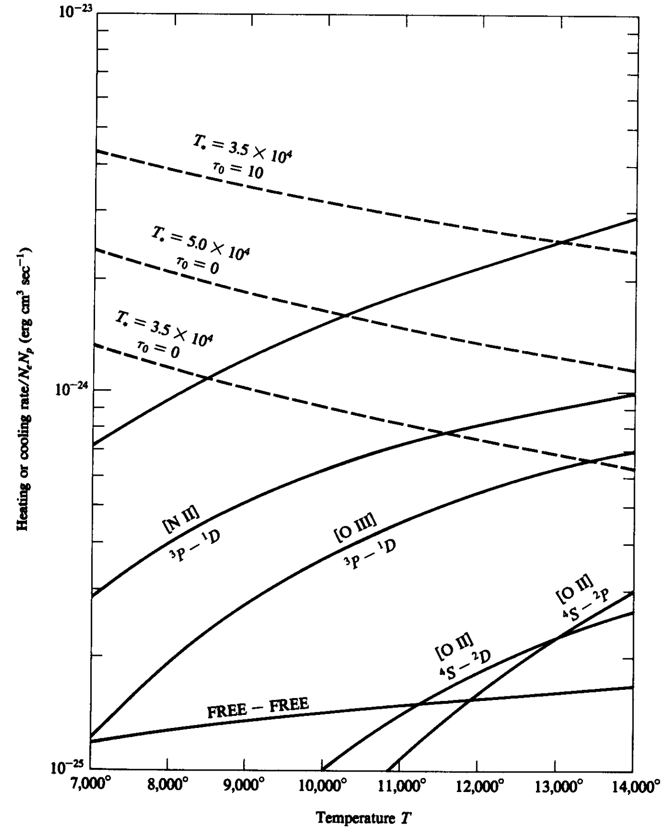


FIGURE 3.3

Same as Figure 3.2, except that collisional de-excitation at $N_e = 10^4$ cm^{-3} has been approximately taken into account in the radiative cooling rates.

Higher density – collisional quenching of the cooling