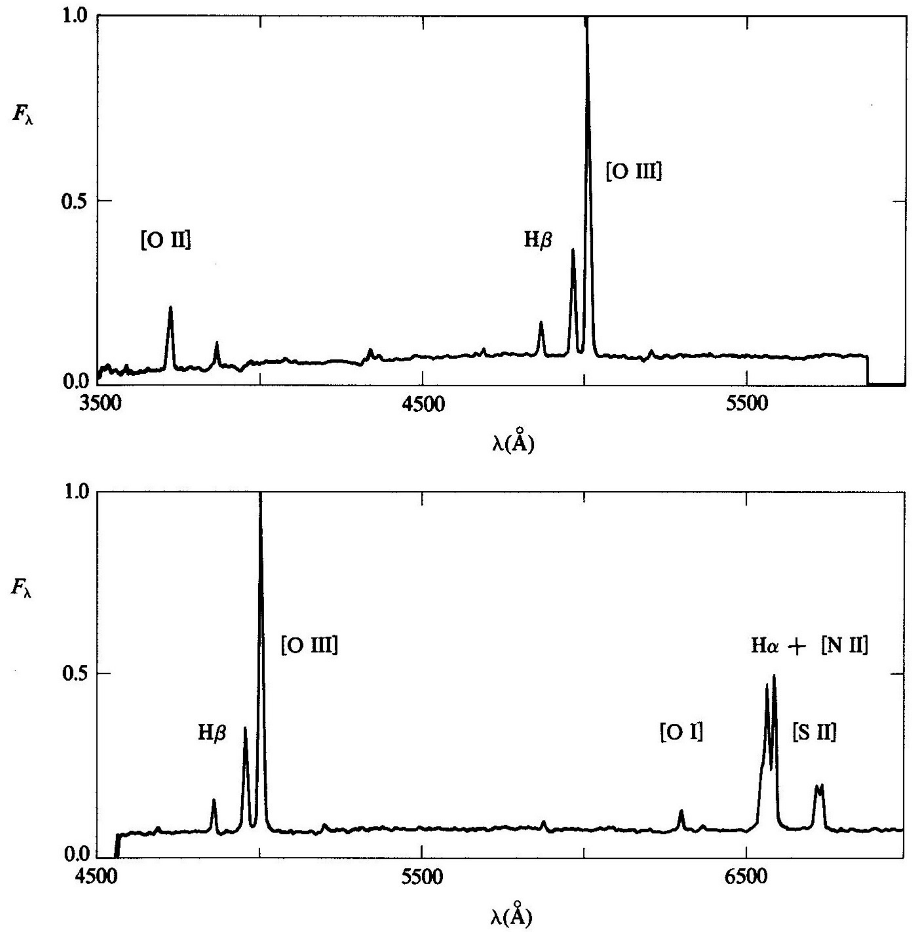
10 - EMITTED SPECTRUM



LYMAN ABSORPTION OPTICAL DEPTH

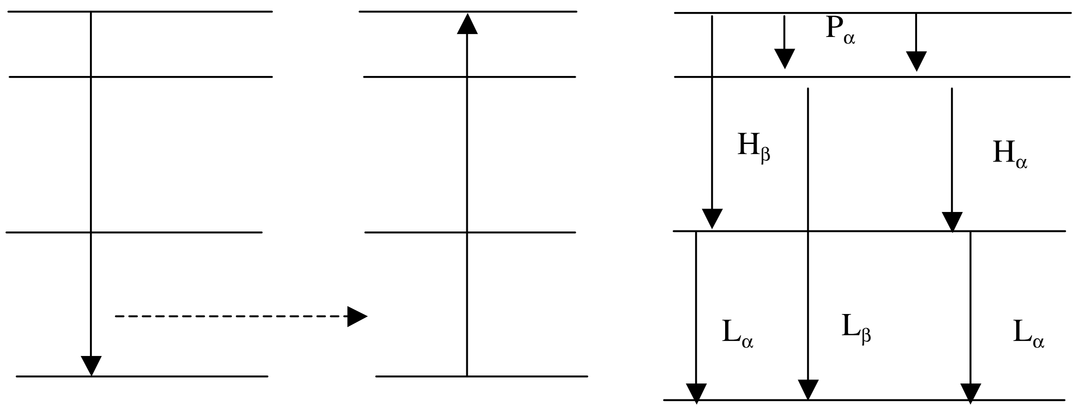
Lyman Continuum Absorption $a_\nu(L_c) = 6.3x10^{-18}\left(\frac{\nu_0}{\nu}\right)^3 cm^2$

Lyman Line Core Absorption $a_\nu \sim 10^{-12} cm^2$

So if τ_{Lc}~1 at the Lyman edge, $\tau_{L\ \text{line core}}$~$10^5$! (up to L_{40})

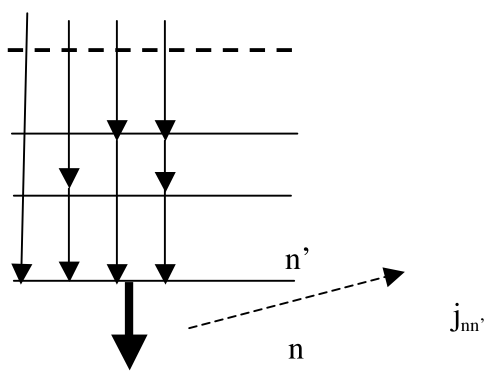
In general, Lyman photons NEVER get out of the ionized region. They just scatter around until they are destroyed.

LINE RADIATION



To get the strengths of other recombination lines, need to calculate the "effective" recombination coefficients – the recombinations to all higher levels that ultimately lead to the population of level n'.

$$n_+ n_e \alpha^{eff} = \frac{4\pi j_{nn'}}{h\nu_{nn'}} \qquad j_{nn'} = emission\ coeff.\ erg\ cm^{-3} s^{-1} Hz^{-1} ster^{-1}$$



= production rate of $\nu_{nn'}$ photons from recombinations and other subsequent cascades from higher levels

Results:

Total Line Strengths:

$$4\pi j(H_\beta) = h\nu_{H\beta} n_4 A_{42}$$
$$= n_p n_e \alpha_{42}^{eff} h\nu_{42}$$
$$\propto n_e^2 T^{-0.8} \quad \textit{approximately}$$

Line Ratios:

$$\frac{j(H_\alpha)}{j(H_\beta)} \propto T^{-0.072}$$

$$\frac{j(H_{15})}{j(H_\beta)} \propto T^{0.033}$$

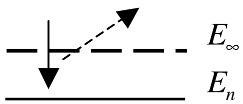
TABLE 4.1

H I recombination lines (Case A)

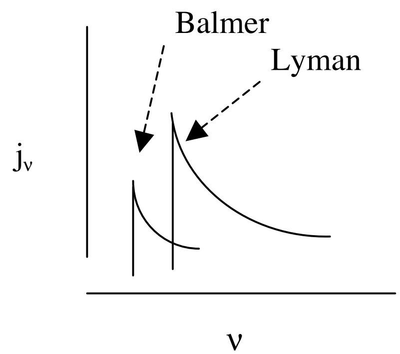
	T			
	2,500° K	5,000° K	10,000° K	20,000° K
$4\pi j_{H\beta}/N_p N_e$ (erg cm^3 sec^{-1})	2.70×10^{-25}	1.54×10^{-25}	8.30×10^{-26}	4.21×10^{-26}
$\alpha_{H\beta}^{eff}$ (cm^3 sec^{-1})	6.61×10^{-14}	3.78×10^{-14}	2.04×10^{-14}	1.03×10^{-14}
Balmer-line intensities relative to Hβ				
$j_{H\alpha}/j_{H\beta}$	3.42	3.10	2.86	2.69
$j_{H\gamma}/j_{H\beta}$	0.439	0.458	0.470	0.485
$j_{H\delta}/j_{H\beta}$	0.237	0.250	0.262	0.271
$j_{H\epsilon}/j_{H\beta}$	0.143	0.153	0.159	0.167
$j_{H8}/j_{H\beta}$	0.0957	0.102	0.107	0.112
$j_{H9}/j_{H\beta}$	0.0671	0.0717	0.0748	0.0785
$j_{H10}/j_{H\beta}$	0.0488	0.0522	0.0544	0.0571
$j_{H15}/j_{H\beta}$	0.0144	0.0155	0.0161	0.0169
$j_{H20}/j_{H\beta}$	0.0061	0.0065	0.0068	0.0071
Lyman-line intensities relative to Hβ				
$j_{L\alpha}/j_{H\beta}$	33.0	32.5	32.7	34.0
Paschen-line intensities relative to corresponding Balmer lines				
$j_{P\alpha}/j_{H\beta}$	0.684	0.562	0.466	0.394
$j_{P\beta}/j_{H\gamma}$	0.609	0.527	0.460	0.404
$j_{P\gamma}/j_{H\delta}$	0.565	0.504	0.450	0.406
j_{P8}/j_{H8}	0.531	0.487	0.443	0.404
j_{P10}/j_{H10}	0.529	0.481	0.439	0.399
j_{P15}/j_{H15}	0.521	0.465	0.429	0.396
j_{P20}/j_{H20}	0.508	0.462	0.426	0.394

CONTINUUM RADATION

H I Free-Bound



$$h\nu = \tfrac{1}{2} mv^2 + \left[E_\infty - E_n\right]$$



$$j_\nu = \frac{1}{4\pi} n_{H^+} n_e \sum_n \sum_L \mathrm{v}\,\sigma_{nL} f\left(\mathrm{v}\right) h\nu \frac{d\mathrm{v}}{d\nu}$$

H I Free-Free

$$j_\nu = \frac{1}{4\pi} n_{H^+} n_e \frac{32 Z^2 e^4 h}{3 m^2 c^3} \left(\frac{\pi h \nu_0}{3kT}\right)^{1/2} e^{-h\nu/kT} g_{ff}$$

$$where \quad g_{ff}(T,Z,\nu) \approx 1 \; at \; "visible" \; wavelengths$$

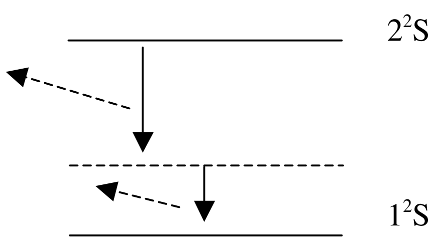
$$FF + FB \quad j_\nu = \frac{1}{4\pi} n_{H^+} n_e \gamma_\nu\left(H^0, T\right)$$

Can also do for He I and He II.

“Two-Photon Process”

Generally, the H atom cannot make the transition from the 2^2S state to the 1^2S state since the photon must carry away some angular momentum. Here QM allows for a “virtual” intermediate state to be used, resulting in TWO photons being emitted.

H I 2γ



$$h\nu_1 + h\nu_2 = h\nu(L\alpha) = h\nu_{12}$$

Probability distribution by number is symmetric about $h\nu_{21}/2$.

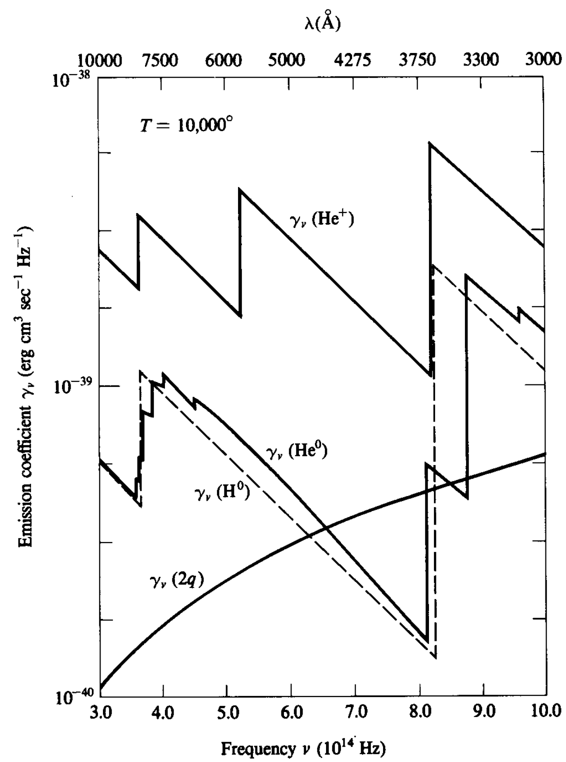
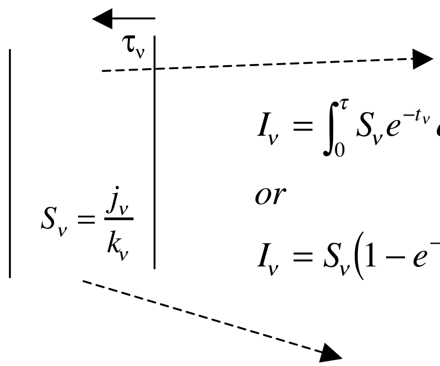


FIGURE 4.1
Frequency variation of continuous-emission coefficient $\gamma_\nu(\mathrm{H}^0)$, $\gamma_\nu(\mathrm{He}^0)$, $\gamma_\nu(\mathrm{He}^+)$, and $\gamma_\nu(2q)$ in the low-density limit $N_e \to 0$, all at $T = 10{,}000°$ K.

Radio Continuum



$$I_\nu = \int_0^\tau S_\nu e^{-t_\nu} \, dt_\nu$$

or

$$I_\nu = S_\nu \left(1 - e^{-\tau_\nu}\right) \quad for \quad S_\nu = const.$$

In TE, $I_\nu = S_\nu = B_\nu \quad (Planck\ fn.)$. Although not strictly TE, because the velocity distribution is essentially maxwellian, we can use $S_\nu = B_\nu$ for the free-free emission.

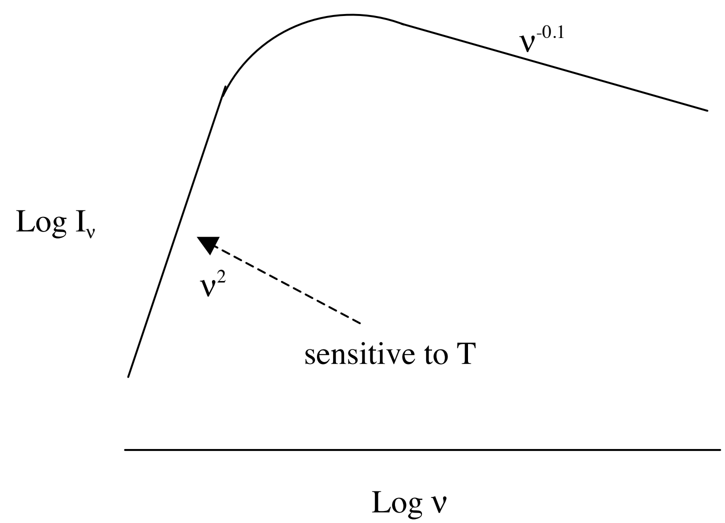
$$I_\nu = B_\nu \left(1 - e^{-\tau_\nu}\right)$$

At radio wavelengths we are in the Rayleigh-Jeans tail of the blackbody curve, so that

$$B_\nu \approx \frac{2\nu^2}{c^2} kT$$

$$For\ \tau_\nu << 1 \quad I_\nu = \frac{2\nu^2}{c^2} kT \tau_\nu \propto \nu_9^{-0.1} T^{-0.35} \ \ insensitive\ to\ T, \nu$$

$$For\ \tau_\nu >> 1 \quad I_\nu = \frac{2\nu^2}{c^2} kT \propto \nu_9^2 T \quad sensitive\ to\ T, \nu$$



DETERMINATION OF T

Because the $^1S_0$ and $^1D_2$ have different energies, they are sensitive to T.

Low n Limit

Every $^3P \rightarrow {}^1D$ collision leads to $\lambda 5007$ or $\lambda 4959$
Every $^3P \rightarrow {}^1S$ collision leads to EITHER $\lambda 2321$
OR
$\lambda 4363$ PLUS [$\lambda 5007$ or $\lambda 4959$]

[in practice, $^3P \xrightarrow[collisional]{} {}^1D$ dominates over $^3P \xrightarrow[collisional]{} {}^1S \xrightarrow[radiative]{} {}^1D$ in populating ^{1}D]

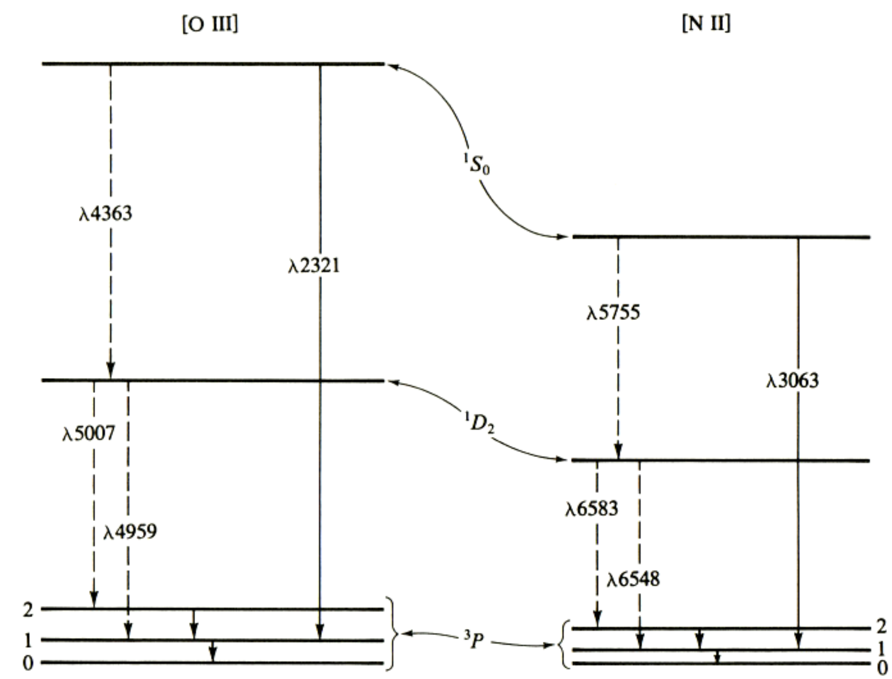


FIGURE 3.1
Energy-level diagram for lowest terms of [O III], all from ground $2p^2$ configuration, and for [N II], of the same isoelectronic sequence. Splitting of the ground ^3P term has been exaggerated for clarity. Emission lines in the optical region are indicated by dashed lines, and by solid lines in the infrared and ultraviolet. Only the strongest transitions are indicated.

For simplicity, let us call the ^{3}P, ^{1}D, and ^{1}S levels, 1, 2, and 3.

$$j(4959)+j(5007)=\frac{n_1 n_e q_{12} h\nu_{12}}{4\pi}$$

$$j(4363)=\frac{n_1 n_e q_{13} h\nu_{23}}{4\pi}\underbrace{\left[\frac{A_{32}}{A_{32}+A_{31}}\right]}_{\substack{\text{fraction out of 3}\\ \text{making } \lambda 4363}}$$

$$q_{12}=\frac{8.63x10^{-6}}{\omega_1 T^{1/2}}\Omega_{12}e^{-\chi_{12}/kT}$$

$$q_{13}=\frac{8.63x10^{-6}}{\omega_1 T^{1/2}}\Omega_{13}e^{-\chi_{13}/kT}$$

$$so,\ \frac{j(4959+5007)}{j(4363)}=\frac{\Omega_{12}}{\Omega_{13}}\left[\frac{A_{32}+A_{31}}{A_{32}}\right]\frac{\nu_{12}}{\nu_{13}}e^{-(\chi_{13}-\chi_{12})/kT}$$

Applying first-order corrections for collisions downward, get

$$\frac{j(4959+5007)}{j(4363)}=\frac{7.73e^{3.29x10^4/T}}{1+4.5x10^{-4}\left(n_e/\sqrt{T}\right)}$$

A similar expression can be obtained for the [N II] lines.

NOTE: λ4363 IS ALWAYS WEAK AND HARD TO MEASURE ACCURATELY.

$$[O\ III]\quad \frac{j(4959+5007)}{j(4363)}\sim 300$$

$$[N\ II]\quad \frac{j(6548+6583)}{j(5755)}\sim 100$$

To do this requires calibrated, linear, high dynamic range light detectors. These did not exist until about 30 years ago (CCDs).

NOTE: THESE NUMBERS ARE ONLY MENT TO BE *REPRESENTATIVE* OF “TYPICAL” VALUES. AS THEY DEPEND ON TEMPERATURE, THEY WILL DIFFER FROM REGION TO REGION!!

TABLE 5.1
Temperature determinations in H II regions

	[N II]			[O III]	
Nebula	$\frac{I(\lambda 6548)+I(\lambda 6583)}{I(\lambda 5755)}$	$T(^\circ\ K)$	$N_e/T^{1/2}$	$\frac{I(\lambda 4959)+I(\lambda 5007)}{I(\lambda 4363)}$	$T(^\circ\ K)$
NGC 1976 2b	81	10,000	51	338	8,700
NGC 1976 1a	102	9,100	68	371	8,500
NGC 1976 5b	111	8,900	21	310	8,900
NGC 1976 5a	189	7,500	12	263	9,300
M 8 I	162	7,900	(10)	445	8,100
M 17 I	257	6,900	(10)	330	8,700
NGC 2467 1a	46	13,000	(1)	129	11,600
NGC 2467 1b	53	12,200	(1)	137	11,400
NGC 2359 av	—	—	(1)	90	13,200

CONTINUUM/LINE ratio has a *weak* sensitivity to T: $\frac{j(4861\ cont)}{j(H_\beta)} \sim T^{0.9}$ *near* $T \sim 10^4 K$

RADIO CONTINUUM: *When* ν *is small and* τ *is large:* $I_\nu = \frac{2\nu^2}{c^2} kT$ *(Rayleigh-Jeans Tail of Planck fn.)*

DETERMINATION OF n_e

If there are excited states with 2 or more levels close in energy, but different A's or q's, one can calculate the density n.

Example – [O II]

Low n Limit

Here, every collision up is followed by a radiative transition down. The rate of the transitions from each level J will be proportional to their statistical weights (2J+1) and not on the value of A_{ij}.

$$\Rightarrow \frac{j(3729)}{j(3726)} = \frac{\left[2\left(\frac{5}{2}\right)+1\right]}{\left[2\left(\frac{3}{2}\right)+1\right]} = \frac{3}{2}$$

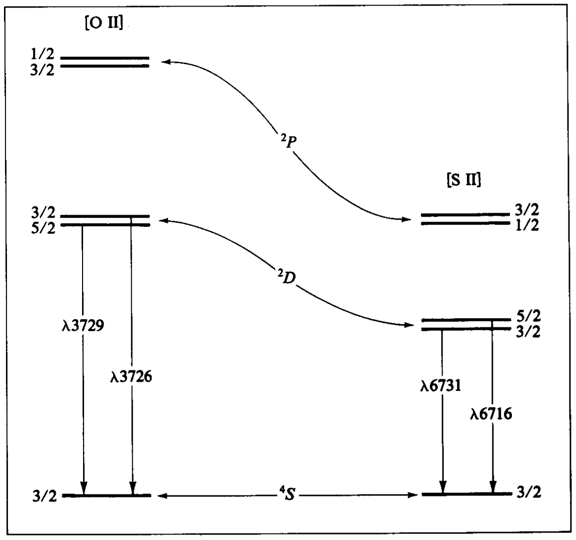


FIGURE 5.2
Energy-level diagrams of the $2p^3$ ground configuration of [O II] and $3p^3$ ground configuration of [S II].

High n Limit

As $n \rightarrow \infty$ the relative populations are in the ratio of their statistical weights, ***and*** the ratio of emission is *that* ***times*** the rate of downward transmission for each level $\propto n_{level}A_{line}$ (downward radiative transitions must compete with downward collisional transitions):

$$\frac{j(3729)}{j(3726)} = \frac{n\left({}^2D_{5/2}\right)A_{3729}}{n\left({}^2D_{3/2}\right)A_{3726}} = \frac{3}{2}\left(\frac{3.6x10^{-5}}{1.8x10^{-4}}\right) = 0.30$$

In between these limits, the ratios are more complicated.

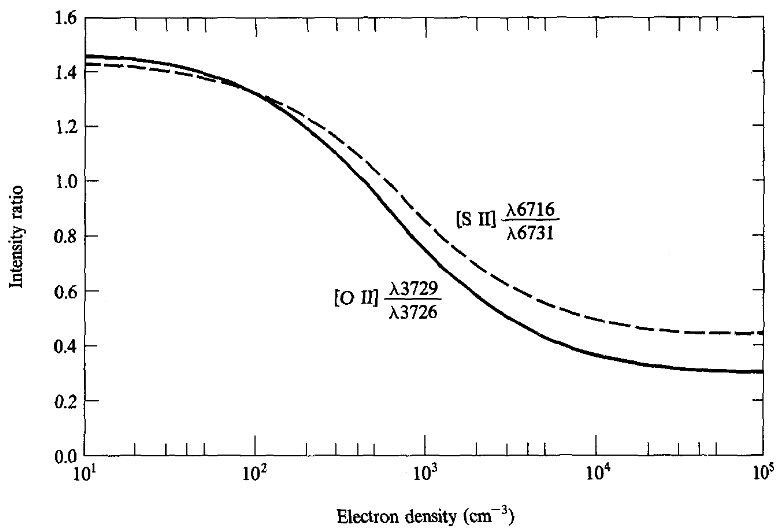


FIGURE 5.3
Calculated variation of [O II] (*solid line*) and [S II] (*dashed line*) intensity ratios as function of N_e at $T = 10,000°$ K. At other temperatures the plotted curves are very nearly correct if the horizontal scale is taken to be $N_e(10^4/T)^{1/2}$.

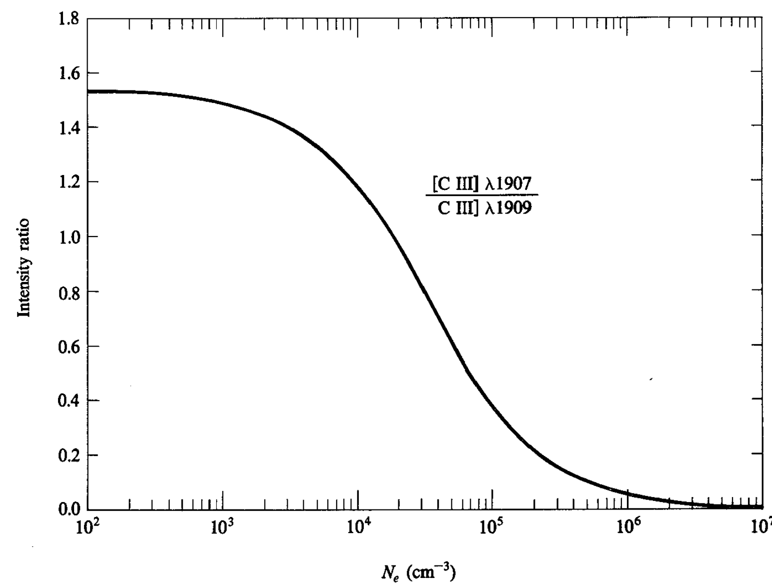


FIGURE 5.5
Calculated variation of [C III] $\lambda1907$/C III] $\lambda1909$ intensity ratio as function of electron density N_e at $T = 10,000°$ K.

The C [III] and C III] lines are in the UV, and used when observing high-redshift galaxies and quasars (or when using Hubble, IUE, etc.).

OTHER ISSUES

Chemical Composition

It is possible to measure the abundance of O, N, etc. versus H along a path s:

$$\frac{I_\nu(ion)}{I_\nu\left(H^+ \rightarrow H^0\ line\right)} = \frac{\nu_{ion\,line}}{\nu_{H\,line}} \frac{\int n_{ion} \frac{8.63x10^{-6}}{T^{1/2}} \frac{\Omega}{\omega} e^{-etc}\, b\, ds}{\int n_{H^+} \alpha_{\substack{rec \\ line}}\, ds}$$

where b is the fraction of excitations to the upper level that result in the line of interest (a branching ratio).

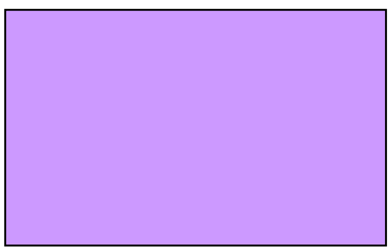
Clumping

Optical forbidden line strengths $\propto n_e$

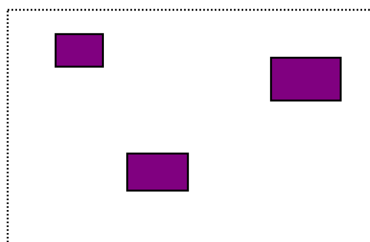
Radio continuum strength (H^+) $\propto n_e^2$

$If \left\langle n_e^2 \right\rangle \neq \left\langle n_e \right\rangle^2 \ then\ n_e(radio) \neq n_e([line])$

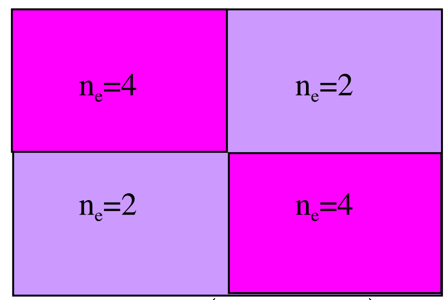
NOT THIS:



BUT THIS:



Example



$$\left\langle n_e \right\rangle = \frac{(4+2+4+2)}{4} = \frac{12}{4} = 3 \quad and \quad \left\langle n_e \right\rangle^2 = 9$$

$$\left\langle n_e^2 \right\rangle = \frac{16+4+16+4}{4} = \frac{40}{4} = 10$$

$$Put\ another\ way, \quad \sqrt{\left\langle n_e^2 \right\rangle} = \sqrt{10} \neq 3 = \left\langle n_e \right\rangle$$