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A THEORY OF ANEURYSM SOUNDS

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Abstract—Narrow-band sounds are known to be associated with some intracranial aneurysms. Previously proposed theories for the mechanism of aneurysm sounds do not satisfactorily explain the small spectral widths of the sounds. A simple theory is proposed here which gives quantitatively correct predictions of the spectral widths and which also explains other salient features of aneurysm sounds. The physical features of the aneurysm are described in terms of lumped mechanical elements, and the interaction between the aneurysm vibration and the blood flow is recognized as having the characteristic features of a nonlinear feedback system. The resulting model, with the application of the method of describing function analysis commonly used in nonlinear control theory, yields predictions of steady oscillation frequencies and predictions of the ranges of arterial flow velocities for which substantial oscillations can be excited. An analysis of radiation losses associated with peristaltic waves indicates that aneurysms, in the absence of any nonlinearity, behave as low-quality factor resonators with resonator quality factors on the order of 1–10, much lower than those that would be inferred from the observed spectral widths of aneurysm sounds. Aneurysm sounds are predicted by the present nonlinear theory to have center frequencies on the order of 400 Hz and bandwidths corresponding to quality factors on the order of 40, in good agreement with *in vivo* observations. It is concluded that linear resonance theories are incapable of fully describing aneurysm sounds. Instead, narrow-band aneurysm sounds are a result of a self-excited nonlinear oscillation that involves the disturbed arterial flow, the flow into and out of the aneurysm, and the expansion and contraction of the aneurysm volume. The finite magnitude of the spectral peaks arises because of the time variation of the arterial flow during the cycle associated with the heartbeat.

Keywords: Aneurysm; Sound; Vibration; Diagnosis; Circle of Willis.

INTRODUCTION

A specific type of sound is known to be associated with the flow of blood past an intracranial saccular aneurysm. The sound is narrow-band, having a 'musical' quality (Ferguson, 1970). Sounds of this type have been detected with air microphones during aneurysm-clipping surgery (Ferguson, 1970), and externally with 'electronic stethoscopes' (Olinger and Wasserman, 1977; Sekhar and Wasserman, 1984; Wasserman, 1975). Typically, aneurysm sounds have center frequencies of the order of 500 Hz and bandwidths corresponding to quality factors (for a given peak of a signal's spectrum, defined as the ratio of the center frequency to the half-power bandwidth) of the order of 30.

Currently, aneurysms are detected by methods such as X-ray and magnetic resonance angiography. X-ray angiography, the best known method of aneurysm detection, involves intra-arterial injection of contrast agents, causing a risk of stroke. Magnetic resonance angiography, in which contrast agents are injected intravenously, is somewhat less sensitive. Since there is as of yet no ideal method of aneurysm diagnosis, and because rupture of intracranial aneurysms results in massive stroke, any safe, noninvasive method of aneurysm detection and characterization has great potential clinical interest.

A number of workers have suggested that aneurysm sounds may be useful for noninvasive diagnosis of intracranial aneurysms (Kosugi *et al.*, 1983; Sclabassi *et al.*, 1987; Sekhar and Wasserman, 1984; Wasserman, 1975). However, clinicians currently do not listen for intracranial aneurysm sounds both because the sounds are low in amplitude when measured externally (though they can be picked up with electronic stethoscopes), and because knowledge of aneurysm sounds existing to date does not provide the capability of distinguishing aneurysm sounds from other arterial sounds. Although some narrow-band sounds in intracranial arteries do result from aneurysms, some may result from other mechanisms (Aaslid and Nornes, 1984a; Allen and Mustian, 1962; Bruns, 1959), so that noninvasive diagnosis using aneurysm sounds is not possible without further knowledge of the detailed characteristics of aneurysm sounds and their distinction from other arterial sounds.

The present paper puts forth the hypothesis that the narrow-band nature of aneurysm sounds results from a nonlinear coupling between aneurysm vibration and unstable arterial flow. The analysis presented here provides a physical explanation the characteristics of aneurysm sounds, including their bandwidths and their time-varying frequencies. The results of the theory may be used to distinguish aneurysm sounds from

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other narrow-band sounds present in intracranial arteries, as is necessary if aneurysms are to be noninvasively diagnosed using their sounds. The results also have application to characterization of aneurysmal tissue, and may lend a better understanding to the process of aneurysmal rupture.

METHODS

Below, the characteristics of the vibrational response of aneurysms are derived in terms of lumped mechanical elements. The mechanism for the excitation of aneurysm sounds is then hypothesized to be a nonlinear coupling between aneurysm vibration and disturbances in the unstable mean flow. This nonlinear coupling is analyzed using the feedback model of Mast and Pierce (1994). The main results of the feedback model are discussed later in this section.

When the oscillatory velocity of the fluid in the opening of the aneurysm is approximated as uniform, one can describe the motion of this slug of fluid as that of a one-dimensional oscillator. Newton's second law leads to the equation describing the response of the oscillator:

$$F(x,t) = M\ddot{x} + R\dot{x} + Kx, \quad (1)$$

where x is the average (inward) displacement of the fluid in the cavity opening, M is an effective system mass, R is the system damping, K is an effective stiffness, and $F(x,t)$ is the external force, which is equal to the pressure outside the opening times the area of the opening. It depends on x because of the nonlinear interaction between the unstable mean flow and the oscillatory flow in the aneurysm's opening. This nonlinear interaction is discussed further later in this section.

The storage of potential energy in the aneurysm sac's tissue is represented by K (the stiffness) in equation (1). The stored potential energy is associated with the stress in the sac, so that the stiffness is proportional to the Young's modulus E of the sac.

Since the walls of aneurysms are thin compared to their radii (Steiger *et al.*, 1989), the stress in the walls can be approximated as two-dimensional (plane) stress. Furthermore, for breathing-mode vibrations there is no preferred direction for the stress. The stiffness K associated with the cavity walls can thus be derived as follows: consider a flexible, spherical, membrane-walled container filled with incompressible fluid. The container has a small circular opening, in which a piston is inserted (Fig. 1). When the piston is displaced a small amount, the container's walls exert a spring-like restoring force. The linear relationship between the piston displacement and the stored potential energy yields the system stiffness K.

The total potential energy stored in the flexible walls of the cavity is

$$\mathrm{PE} = \oint \frac{\sigma^2 h}{2E} \mathrm{d}S, \quad (2)$$

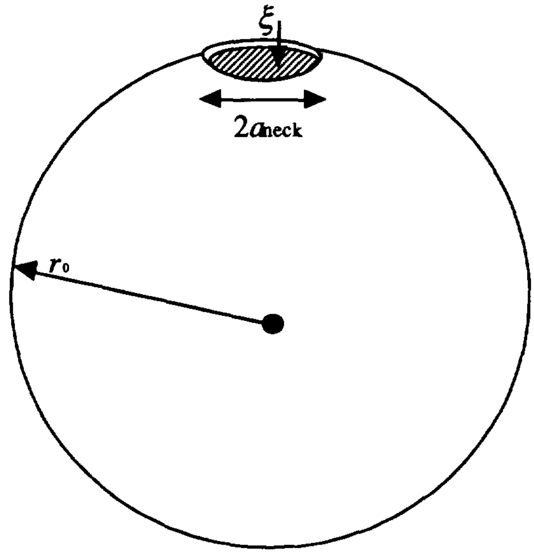

Fig. 1. Spherical aneurysm with a small opening. The piston inserted into the opening has displacement ξ.

where h is the wall thickness, E is the Young's modulus of the aneurysmal tissue, σ is the stress at a point in the wall and dS is a differential area element of the cavity wall.

For a uniformly loaded shell in breathing-mode displacement, the strain is the same throughout the shell walls and is given by

$$\varepsilon = \frac{\xi A}{S r_0}, \quad (3)$$

where A is the area of the neck, S is the surface area of the sac, and ξ is the piston displacement.

For small displacements (such that linear elasticity applies), the stress is simply $\sigma = E\varepsilon$, where E is the Young's modulus of the aneurysmal tissue. Thus, the total potential energy stored in the cavity walls is, from equation (2),

$$\mathrm{PE} = \frac{EhA^2\xi^2}{2Sr_0^2}. \quad (4)$$

For a one-dimensional oscillator, the stored potential energy relates to the stiffness parameter K as

$$\mathrm{PE} = \frac{1}{2}K\xi^2, \quad (5)$$

(Morse, 1981, p. 23) so that one identifies the stiffness associated with the cavity walls as

$$K = \frac{EhA^2}{2Sr_0^2}. \quad (6)$$

Another basic parameter descriptive of the vibratory response of a lumped-element oscillator is the inertance, or mass, associated with the system kinetic energy, which is concentrated in the vicinity of its opening. This mass, M_{blood}, can be estimated using an inner 'end correction' which represents the amount of fluid in the plug oscillating in the neck (Pierce, 1989, pp. 348–349):

$$M_{\text{blood}} \approx \rho A L', \tag{7}$$

where L' is about 0.8 times the radius of the opening. Only an inner end correction is used here because aneurysms in the blood stream are exposed to a grazing mean flow. End corrections are known to decrease with increasing mean-flow velocity (or decreasing Strouhal number, for constant frequency). When the travel time across the aneurysm neck for a fluid particle in the free stream becomes comparable to an acoustic period, less fluid can be carried back and forth with the oscillatory flow in the neck. The outer end correction is then effectively 'blown away' (Ronneberger, 1972; Panton and Miller, 1975), and the appropriate end correction is one half that for an orifice flanged on both sides, or $L' = 0.8a_{\text{opening}}$. Thus, the appropriate expression for the mass is

$$M_{\text{blood}} \approx 0.8 \rho a A. \tag{8}$$

The kinetic energy associated with M_{blood} is

$$\text{KE} = \frac{1}{2} M_{\text{blood}} \dot{\xi}^2. \tag{9}$$

The natural frequency of an aneurysm (the frequency of free vibrations) is that frequency at which, for a sinusoidal oscillation of the form $\xi = \xi_{\max} e^{-i\omega t}$, the maximum stored potential energy in the walls of the sac [given by equation (4)] exactly balances the maximum kinetic energy of the fluid oscillating in the aneurysm's opening [given by equation (9) with $\dot{\xi}$ replaced by $\omega \xi_{\max}$]. Setting the potential energy equal to the kinetic energy then yields

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{2EAh}{\rho L' S r_0^2}}. \tag{10}$$

This is identical to the result of Wasserman (1975) for the case where the aneurysm's wall has a Poisson's ratio of 1/2, that is, when aneurysmal tissue is taken to be incompressible.

The last parameter needed to specify the response of a lumped-element oscillator is a damping parameter [R in equation (1)] associated with the system's resistance, which serves to limit the amplitude of the oscillator's response near its natural frequency. For constant-frequency oscillations at the natural frequency of the oscillator, this resistance is given by

$$R = \frac{F_{\text{neck}}}{u}, \tag{11}$$

where F_{neck} is the force applied to the slug of fluid oscillating in the aneurysm opening and u is the average velocity in the opening.

This resistance element is due to a radiation resistance (it will be shown below that this radiation resistance dominates over the viscous dissipation in the opening or neck of the aneurysm). The 'radiation' of interest is due to fluid-elastic waves, which carry energy away from the aneurysm along the artery. These are peristaltic (breathing-type) waves which travel at the usual pulse wavespeed [for a detailed discussion, see Fung (1984)]. The blood can still be taken to behave incompressibly, since the propagation speed of peristaltic waves is much less than the sound speed in blood. Peristaltic waves travel at a speed approximately given by the Moens–Korteweg wavespeed,

$$c = \sqrt{\frac{Eh}{\rho d}}, \tag{12}$$

where E is the Young's modulus of the arterial wall, h is the arterial wall thickness, d is the arterial diameter, and ρ is the density of blood. Values for these parameters measured *in vitro* are $E = 2.5 \times 10^6$ N m^{-2} (Steiger *et al.*, 1989) and $h \sim 1 \times 10^{-4}$ m -2×10^{-4} m (Scott *et al.*, 1972). In the present study, the wall thickness was assumed to be 1.5×10^{-4} m. For arteries in the size range of the intracranial arteries, the peristaltic wavespeed c is on the order of 10 m s^{-1} (Caro *et al.*, 1974), while the sound speed of blood is about 1500 m s^{-1}.

Now consider a vessel with an aneurysm (Fig. 2). There is an oscillating flow in the aneurysm's opening having volume velocity $q_{\text{neck}} = uA_{\text{neck}}$ where u is the average velocity in the opening of the aneurysm. Conservation of mass requires that the volume velocity exiting the control volume be equal to q_{neck}. For a symmetrical configuration (as in Fig. 2), the volume velocity exiting either end of the control volume is

$$q_{\text{vessel}} = \frac{q_{\text{neck}}}{2} = \frac{uA_{\text{neck}}}{2}. \tag{13}$$

The pressure associated with the peristaltic waves is given by

$$p = \frac{q_{\text{vessel}} \rho c}{A_{\text{vessel}}}, \tag{14}$$

where ρ is the density of blood and c is the peristaltic wavespeed.

The force on the aneurysm's opening is $F_{\text{neck}} = pA_{\text{neck}}$, so that the result for the radiation resistance is

$$R_{\text{rad}} = \frac{F_{\text{neck}}}{u_{\text{neck}}} = \frac{\rho c A_{\text{neck}}^2}{n A_{\text{vessel}}}, \tag{15}$$

where n is the number of possible peristaltic-wave paths. For an aneurysm in a straight artery section, as shown in Fig. 2, n is equal to 2. For an aneurysm arising at a bifurcation, n is equal to the number of branches (both upstream and downstream) at the point of the aneurysm's formation.

A useful parameter describing a lumped-element oscillator's damping characteristics is the 'quality factor' Q. This parameter represents the ratio of the total energy within the oscillator to the energy lost per radian when the system is at resonance (Pierce, 1989,

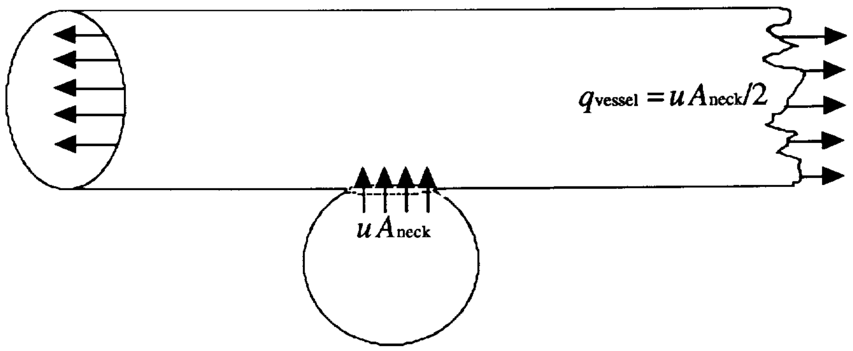

Fig. 2. Schematic illustrating the derivation of the radiation resistance associated with peristaltic waves.

p. 122). The quality factor of an oscillator is given by

$$Q = \frac{\sqrt{KM}}{R}. \quad (16)$$

Q is also approximately equal to the normalized bandwidth of the oscillator's admittance curve:

$$Q \approx \frac{f_0}{f_2 - f_1}. \quad (17)$$

Here f_0 is the natural frequency of the oscillator, and f_1 and f_2 are the frequencies of the 'half-power points' for which the response $x = x_{max}/\sqrt{2}$. When a resonator is excited by broadband, sound, the $Q_{measured}$ observed on a spectral analyzer will be that Q given by equation (17), which will be the same as the Q calculated from equation (16).

The mechanism of aneurysm sounds is hypothesized here to be a self-excited oscillation due to a nonlinear coupling between the aneurysm vibration and the unstable mean flow in the intracranial arteries. The nonlinear coupling has been analyzed by Mast and Pierce (1995) [see also Mast (1993)] for the general problem of flow excitation of lumped-element resonators such as Helmholtz resonators. In their study, analysis of the available experimental evidence led to the assumption that the coupling occurs as follows: vortices are shed from the upstream edge of an aneurysm's opening at the frequency of the aneurysm's vibration. The vortices are shed with a specific phase relationship to the oscillating flow in the aneurysm's opening. As a vortex is convected past the downstream edge of the opening, it creates a pressure disturbance which forces the aneurysm into further vibration.

The system is nonlinear because the flow disturbance depends on the aneurysm vibration while the aneurysm vibration also depends on the flow disturbance. Although such a system is difficult to treat analytically, the frequency and amplitude of limit cycle oscillation can be found describing function analysis. This is a mathematical technique in which the parts of the system are represented by 'describing functions', analogous to transfer functions, and the product of the describing functions (analogous to the total gain around a feedback loop) is set equal to 1. This analysis has been performed by Mast and Pierce (1994). The results of the analysis are briefly presented here.

The criterion for limit-cycle oscillations of the flow-aneurysm system is

$$\left(\frac{q_o(q_r)}{q_r}\right)\left(\frac{q_r(q_o)}{q_o}\right) = \frac{\beta A^2 \rho U^2}{\omega M |q_r|} e^{-i(3\pi/2 - 2S)} \times \frac{1 - i\Omega/Q}{\Omega^2 - (1 - i\Omega/Q)} = 1. \quad (18)$$

Here the total volume velocity in the aneurysm opening has been split into two parts. The 'orifice flow' q_o is proportional to the pressure disturbance above the opening:

$$q_o = \frac{p_{above}}{-i\omega M}. \quad (19)$$

The 'resonator flow' q_r is the remainder of the volume velocity in the opening:

$$q_r = q_{total} - q_o. \quad (20)$$

S is the Strouhal number $\omega d/U$ based on the radial frequency of oscillation ω, the streamwise opening diameter d, and the free-stream velocity U. Q is the quality factor of the aneurysm and Ω is the normalized frequency ω/ω_0. Equation (18) can be solved numerically to find the amplitude and frequency of oscillation.

The requirement that the imaginary part of the loop gain must be zero leads to the dispersion relation

$$2S = \tan^{-1}\left(\frac{Q^2 - \Omega^2(Q^2 - 1)}{\Omega^3 Q}\right) + 2n\pi, \quad (21)$$

where n is an integer.

When a particular value of U is specified, equation (21) can easily be solved numerically for ω, yielding the frequency of oscillation. When the resulting value of ω is substituted into equation (18), one obtains a purely real equation for the 'resonator' flow q_r. The total velocity response in the neck of the aneurysm can then be calculated as

$$|q_{\text{total}}| = |q_r| \left| 1 + \left(\frac{q_r(q_o)}{q_o} \right)^{-1} \right|. \quad (22)$$

Equation (18) may be used to calculate the 'instantaneous' frequency and amplitude of an aneurysm's self-excited oscillation. Since the mean flow in the intracranial arteries varies slowly with time, having a periodicity associated with the heartbeat cycle, the amplitude and frequency of aneurysm oscillation also vary slowly with time.

The measured signal from an aneurysm oscillation can thus be approximated as a quasi-sinusoid with varying amplitude $x_0(t)$ and radial frequency $\omega(t)$ determined from equation (18). The signal is then

$$x(t) = x_0(t)e^{-i\phi(t)}, \quad (23)$$

where the phase $\phi(t)$ is given by

$$\phi(t) = \int_{t_0}^{t} \omega(\tau)\mathrm{d}\tau. \quad (24)$$

Flow disturbances and narrow-band murmurs of the intracranial arteries have been observed to occur preferentially during the portion of systole in which the mean arterial flow is decelerating (Aaslid and Nornes, 1984a). It will be assumed here that the mean arterial flow is decelerating during the time period in which the aneurysm oscillations of interest occur. Envelopes of the mean-flow velocity in the intracranial arteries are shown in the Doppler studies of Aaslid *et al.* (1984b) and Sekhar *et al.* (1988). These studies show, in the middle cerebral artery, deceleration consistent with the formula

$$U = U_{\text{peak}} - \left(\frac{U_{\text{peak}}}{2} \text{s}^{-1} \right) t. \quad (25)$$

Using this envelope for the mean-flow velocity signal, envelopes of the resulting frequency and amplitude of oscillation were calculated. The resulting signals were Hanning windowed (Oppenheim and Schafer, 1989) with a window length of 0.25 s as was done by Wasserman (1975), so that the signals' spectra could be compared with Wasserman's experimental results. They were then Fourier transformed to determine the characteristics of the spectra which would be observed on an experimental investigator's frequency analyzer.

From the Fourier transform, one can determine a quality factor Q_{measured} to quantify the bandwidth of any narrow-band components of $\hat{x}(\omega)$. Q_{measured} is not the same as the aneurysm's quality factor $Q = \sqrt{KM}/R$, but is given by the half-power points of the peak being examined:

$$Q_{\text{measured}} = \frac{f_{\text{peak}}}{f_2 - f_1}. \quad (26)$$

Here f_{peak} is the frequency of the peak measured response. The frequencies f_1 and f_2 are those for which the measured response is $|\hat{x}| = |\hat{x}_{\text{peak}}|/\sqrt{2}$.

The methods discussed above were used to obtain results for the characteristics of self-excited oscillations of aneurysms subjected to a grazing flow. The methods were applied to a number of aneurysms pictured in the literature. In most cases, the specific dimensions of the aneurysms and vessels were not given. Thus, to obtain numerical results, the dimensions of most of the aneurysms were estimated based on available information regarding typical dimensions of the intracranial arteries (Blinkov and Glezer, 1968; Truscott, 1955). The Young's modulus E of aneurysmal tissue was taken to be 1.7×10^6 N m^{-2} as measured (*in vitro*, using strips of aneurysmal tissue) by Steiger *et al.* (1989). The aneurysm wall thickness h was taken to be $0.12D$, also as measured by Steiger *et al.* (1989). The peristaltic wavespeed c was calculated using equation (12). The density ρ of blood was taken to be 1060 kg m^{-3} (Spector, 1956, p. 51). Calculation of signal characteristics was performed only for the aneurysms with $Q > 1$, since those with very small Q cannot be expected to incur self-excited oscillation.

Analogous calculations of signal characteristics were performed for signals having instantaneous frequencies directly proportional to the mean-flow velocity. The instantaneous frequencies and amplitudes were taken to be those expected for periodic vortex shedding having frequency within the range of frequencies calculated for the aneurysm sounds. For periodic vortex shedding by a bluff body, the instantaneous frequency is proportional to U, while the instantaneous amplitude is proportional to U^3 (Etkin *et al.*, 1957).

RESULTS

In this section, the results of the theory and methods discussed above are given. The theory for the vibrational response of aneurysms has a number of results which provide some insight into the nature of aneurysm vibration. The feedback theory for the mechanism of aneurysm sounds provides results which quantitatively characterize aneurysm sounds.

The natural frequencies f_0 of aneurysms calculated using the present model range from about 100 to 600 Hz. The quality factors Q range from less than 1 to about 7 (Table 1).

The lumped-element model also qualitatively shows the influence of aneurysm properties on the vibrational characteristics of aneurysms. The natural frequency of an aneurysm increases with the Young's modulus of the aneurysmal tissue, so that aneurysms partly composed of stiff tissue (e.g. thrombus) will have higher natural frequencies. Equation (10) also has the result that the natural frequency of an aneurysm increases with the size of its opening and decreases with the volume of its sac.

The value of the quality factor Q is strongly dependent on the size of the vessel to which the aneurysm is

Table 1. Natural frequencies f_0, lumped-element quality factors Q, and expected mean-flow velocities U_n for peak excitation in flow disturbance modes $n = 1$ ($\lambda = d$) and $n = 2$ ($\lambda = d/2$). Sources: (a) Bassett (1954), (b) Crawford (1959), (c) Stehbens (1963)

No.	d(mm)	f_0(Hz)	Q	$Q_{measured,1}$	$Q_{measured,2}$	U_1(m s^{-1})	U_2(m s^{-1})	Source
1.	2	286	2.4	38	30	1.1	0.57	a, Fig. 1
2.	1	305	5.1	52	39	0.61	0.30	b, Fig. 1
3.	1	430	3.1	45	32	0.86	0.43	b, Fig. 2
4.	6	95	0.54	—	—	—	—	b, Fig. 6
5.	3	371	3.2	43	32	2.2	1.1	b, Fig. 11
6.	2	455	3.9	47	33	1.8	0.91	b, Fig. 13
7.	6	127	0.44	—	—	—	—	c, Fig. 5
8.	1.2	637	6.6	65	46	1.5	0.76	c, Fig. 9
9.	3	190	3.0	42	30	1.1	0.57	c, Fig. 10
10.	6	127	0.54	—	—	—	—	c, Fig. 15

appended. Since the peristaltic wavespeed c goes as $1/\sqrt{d_{artery}}$, the resistance given by equation (15) goes as $d_{artery}^{5/2}$. Thus, the dimensions of a vessel also have a large effect on the vibrations of an aneurysm appended to the vessel. The quality factor also increases with the stiffness of the aneurysm wall, so that an accumulation of thrombus will increase Q.

Equation (18) has the consequence that a resonator such as an aneurysm is prone to self-excited oscillation near its natural frequency when exposed to a mean flow of velocity U_n such that $U_n \approx 2f_0d/n$. The highest-amplitude oscillations occur for $n = 1$. The mode $n = 1$ corresponds to a distance between vortices equal to the streamwise length of the resonator opening d; the mode for which $n = 2$ corresponds to a distance between vortices equal to $d/2$. In general, the wavelength of the flow disturbance is d/n. Each of the U_n can be taken as an approximation of the flow velocity for which peak excitation will occur given a flow disturbance of order n. The calculated velocities U_1 and U_2 (Table 1) range from 0.3 to 2.2 m s^{-1}.

The signal characteristics calculated using the feedback model have the significant result that $Q_{measured}$, the quality factor based on the observed bandwidth of an aneurysm sound, is typically much greater than the quality factor Q of the aneurysm. A typical admittance curve of an aneurysm (shown for aneurysm no. 2 in Fig. 3), which corresponds to its response to external forcing, is quite broadband. Frequency analysis of the sound made by this aneurysm, however, yields a very narrow-band curve (Fig. 4). In each case calculated here (Table 1) $Q_{measured,1}$ (for excitation in the flow disturbance mode for which the flow disturbance wavelength is approximately equal to the streamwise dimension of the opening) is about 10 times the Q of the aneurysm. The observed quality factor $Q_{measured,2}$ for excitation in the mode $\lambda \approx d/2$ is in all cases somewhat less than $Q_{measured,1}$, but still considerably larger than Q.

The quality factor Q of an aneurysm still is correlated with the observed quality factor $Q_{measured}$. This correlation appears because aneurysms of higher Q exert more frequency control on the aneurysm–flow system. A plot of the dispersion relation given by equation (21) (Fig. 5) shows that for a Q of 100, the frequency of oscillation is nearly constant over a wide range of Strouhal numbers, while for a Q on the order of 1 the frequency of oscillation depends strongly on the Strouhal number (and thus on the mean-flow velocity). Resonators with Qs of one or less will not exert much frequency control on flow disturbances to the mean flow; such resonators thus cannot be expected to incur self-excited oscillation in the presence of an unstable mean flow. Thus, aneurysms 4, 7, and

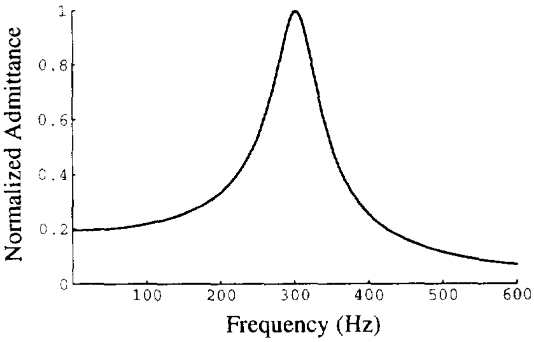

Fig. 3. Admittance for aneurysm no. 2, corresponding to the aneurysm's response to broadband forcing. The bandwidth Q is 5.1.

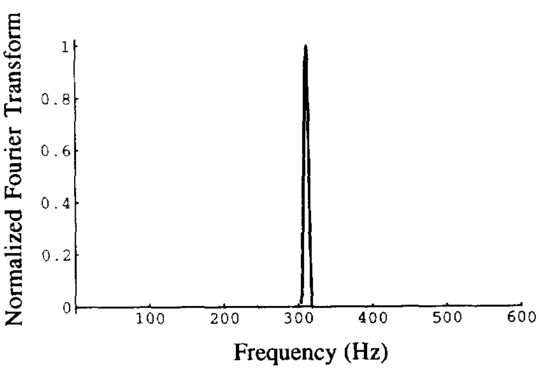

Fig. 4. Observed spectral content for excitation by time-varying flow for aneurysm no. 2. The bandwidth $Q_{measured}$ is 52, an order of magnitude higher than that of the aneurysm's admittance curve.

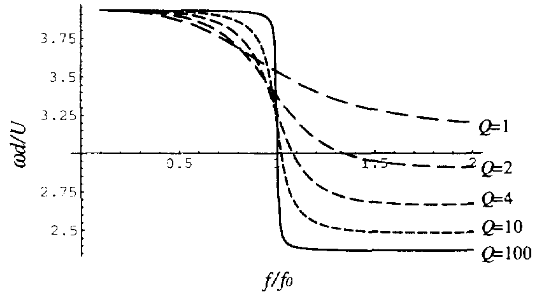

Fig. 5. Plot of nondimensional 'Strouhal' frequency $\omega d/U$ vs sounding frequency f/f_0 for flow-disturbance mode $n = 1$ ($\lambda \approx d$). The curves show that with increasing aneurysm Q, the aneurysm exerts more control on the frequency–velocity dependence of the oscillation, seen here as a steepening of the central portion of the curves.

10 (in Table 1) are not expected to oscillate in the manner discussed in this paper.

The frequency control caused by the presence of an aneurysm having sufficiently high Q causes aneurysm sounds to have considerably narrower bandwidth than sounds caused solely by fluid-mechanical effects. Calculated quality factors for signals associated with periodic vortex shedding were on the order of 20 for mean-flow velocities between 0.5 and 4.0 m s^{-1}. This is considerably smaller than the calculated values of $Q_{measured}$ for aneurysm sounds, which ranged from 30 to 65 (Table 1).

DISCUSSION

The theory presented in this paper provides a physical explanation for the observed characteristics of aneurysm sounds, including bandwidths, frequencies, and ranges of mean-flow velocities for excitation. The goal of the research reported here was to provide such an explanation by analyzing the basic physical processes of aneurysm–flow interaction in as simple a manner as possible, but without oversimplification.

Some earlier studies, in which aneurysms have been modeled as fluid-filled shells with a number of possible resonant modes, yielded natural frequencies within the same range as those predicted by a lumped-element model (Simkins and Stehbens, 1973; Hung and Botwin, 1975). Either approach yields order-of-magnitude agreement with experimental observations on the frequencies of aneurysm sounds. However, the more complicated shell-theory approaches yield little or no additional information to that provided by simple lumped-element models. This is partly due to the fact that real aneurysms are not perfectly spherical in shape and have anisotropic walls. In addition, experimentally observed frequencies of vibration are within the range of frequencies associated with the lowest-order resonance for a fluid-filled shell with an opening, as calculated by Hung and Botwin (1975). This lowest-order resonance, in which the system's compliance is associated with breathing motion of the walls, is physically identical to that predicted by a lumped-element model. In confirmation of this fact, a recent *in vitro* study (van Bruggen, 1994) experimentally verified that the resonance frequencies of aneurysm vibration agree with those predicted by a lumped element model. For these reasons, the lumped-element approach was taken in the present study. The present model goes further than those previously posed (Wasserman, 1975; van Bruggen, 1994) by including a general analysis of the system damping associated with peristaltic waves and the effect of this damping on the characteristics of resonant and self-excited oscillations.

Any model using lumped elements necessarily includes a number of simplifying assumptions. Here, in the derivation of the mass element, the kinetic energy of the system was assumed to be concentrated in the vicinity of the aneurysm's opening, where the fluid velocity associated with the aneurysm vibration is largest. The derivation of the stiffness included the assumptions of spherical shape, uniform composition, and linear elasticity. While aneurysms are not spherical or uniform, the results of lumped-element models are not strongly dependent on the shapes and detailed compositions of the physical systems being modeled, so that the results of the present paper are expected to be valid for nonspherical, nonuniform aneurysms. The assumption of linear elasticity was made based on the experimental results of Steiger *et al.* (1989), who showed that aneurysmal tissue behaves linearly for strains up to about 0.1. Strains as large as 0.1 are not predicted by the present model.

It was also assumed that the radiation resistance associated with peristaltic waves is the dominant loss mechanism in aneurysm vibration. This assumption is valid because aneurysms do not in general have long necks, so that the fluid oscillating in the opening of the aneurysm does not incur significant viscous resistance. The treatment of peristaltic waves assumed that the waves propagate essentially one-dimensionally and that their amplitude is sufficiently small for a linear propagation model to be appropriate. These assumptions are expected to be valid for arteries such as the intracranial arteries (Fung, 1984).

The assumptions of the feedback model also merit some discussion. The model assumes that the mean flow in the intracranial arteries is unstable, so that vortical disturbances arise in response to aneurysm vibration. This assumption is valid while the flow is decelerating, during systole. While the mean flow is decelerating, the boundary layers at the arterial walls tend to develop inflectional profiles which destabilize the mean flow (Sarkpaya, 1966). During the deceleration part of systole, vortical flow disturbances have been observed *in vitro* in models of the intracranial arteries (Ferguson, 1970; Roach *et al.*, 1972) and *in vivo* by Doppler velocimetry (Aaslid and Nornes, 1984a). Aneurysm sounds occur preferentially during systole (Ferguson, 1970; Wasserman, 1975). For these reasons, the deceleration phase of systole was used in the calculations outlined in Methods. The model for the aneurysm–flow interaction also implicity includes the assumption that the mean flow primarily goes past the aneurysm's opening, rather than into the sac. This assumption is valid for laterally attached berry aneurysms and for some aneurysms occurring at bifurcations. However, it is likely not to be valid for many other aneurysms placed at bifurcations. The oscillations of these aneurysms, if they occur, are due to a feedback mechanism somewhat different than that stated in the present paper.

The natural frequencies of many aneurysms, as calculated by the present model, are within the range of observed frequencies of the sounds associated with aneurysms *in vivo*, about 200–800 Hz (van Bruggen, 1994; Ferguson, 1970; Sekhar and Wasserman, 1984; Wasserman, 1975). However, none of the reports based on these experimental studies included data on the sizes of the aneurysm openings, so that precise comparison of frequency results is not possible.

The calculated quality factors Q_{measured} (Table 1) are in order-of-magnitude agreement with those determined by Mast (1993) from the aneurysm-sound spectrograms published by Wasserman (1975). Wasserman's spectrograms, made using signals measured noninvasively with an electronic stethoscope, correspond to values of Q_{measured} within the range of 22–65 for patients diagnosed with aneurysms. Also, as seen in the present calculations, Wasserman's spectrograms show larger values of Q_{measured} for larger center frequencies of the aneurysm sounds. The high predicted values of Q_{measured} are also consistent with the *in vivo* observations of Ferguson (1970), Olinger and Wasserman (1977), and Kosugi *et al.* (1983), More recently, van Bruggen (1994) has reported 'noisy bruits' with spectra corresponding to considerably smaller values of Q_{measured} for aneurysm sounds recorded during surgery. The reason for this discrepancy with previous results is not clear, although it may be related to the large number of spectral averages performed or to the absence of abnormally heightened arterial flow velocities, which were intentionally avoided in van Bruggen's measurements of aneurysm sounds.

The inclusion of damping in the present model allows the observation that aneurysms' narrow-band oscillations are not due to a resonance phenomenon caused by external forcing, either by the pulse waves associated with the heartbeat (Austin, 1971, 1974; Cronin, 1973, 1974; Jain, 1963), or by turbulent blood flow (Ferguson, 1970). All the aneurysms considered in this paper have quality factors Q considerably smaller than observed *in vivo* quality factors Q_{measured} (Table 1). For this reason, the narrow bandwidths of aneurysm sounds are explained only by a self-excited oscillation, that is, the force which acts to excite the aneurysm vibrations must depend on the motion of the fluid in the aneurysm itself. The mechanism for such a force in flow-excited oscillations is a nonlinear coupling between the flow associated with the aneurysm vibrations and an unstable mean flow. This nonlinear coupling can be interpreted as a feedback process, as in the present paper.

The dependence of sounding frequency on mean-flow velocity is much weaker for an arterial sound associated with the self-excited oscillations of an aneurysm than for sounds associated with purely fluid-mechanical mechanisms. Thus, measurement of bruit frequency taken simultaneously with Doppler measurement of the mean-flow velocity in the artery is a potential technique for noninvasive diagnosis of intracranial aneurysms. For those arteries inaccessible to Doppler measurements, spectral peaks of very large Q_{measured} may indicate the presence of aneurysms.

When the internal dimensions of the aneurysm are measured (for instance, by X-ray angiography or magnetic resonance angiography), values of Q and f_0 (determined from the present theory based on the sound's center frequency and frequency–velocity dependence) for the aneurysm could be used to determine average properties of the aneurysmal tissue, such as the average Young's modulus and wall thickness. These parameters could be interpreted to estimate the state of deterioration of an aneurysm's wall, the amount of thrombus (clot) in an aneurysm, and the chances of rupture of the aneurysm.

To sum up, the present theory of aneurysm sounds, which incorporates a lumped-element model for the vibrational response of aneurysms and a nonlinear feedback model for the excitation of aneurysm oscillations, explains the chief characteristics of narrow-band aneurysm sounds and may be used to estimate the characteristics of the sounds based on the aneurysm and artery dimensions and the mean-flow characteristics. The theory has the implications that aneurysms have natural frequencies within the range of observed aneurysm sounds, but that aneurysms are too highly damped to make narrow-band sound as resonators. The observed narrow bandwidths of aneurysm sounds are quantitatively explained by the nonlinear coupling between aneurysm vibration and arterial flow disturbances, which can be considered a self-excited oscillation of the aneurysm–flow system. The narrow bandwidths of aneurysm sounds are explained as due to the weak variation of self-excited oscillation frequency which occurs with the time-varying arterial flow. The theory and its results have application to noninvasive acoustic diagnosis and characterization of aneurysms.

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