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The Canons of Proclus

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1. Ammonius

The Canons of Proclus. In one of the two passage in which Ammonius explicitly attributes a doctrine to his teacher, he describes what he calls "the canons of Proclus." The these refer to a class of formally uniform inferences which in modern terms we would view as instances of a single inference rule.

The doctrine appears to be a development of remarks of Aristotle's on transforming propositions into equivalents that contain negations, *e.g.* the famous line from the start of the *Prior Analytics* which has been taken as the basis for the principle *de omni et nullo*: "We use the expression *predicated of every* when none of the subject can be taken of which the other term cannot be said, and we use *predicated of none* likewise."[1] The generalization reported by Ammonius dictates that *y is true of all x* is equivalent to *not y is true of*

[1] *Prior Analytics* 24b29. This is the text in Aristotle that best approximates the scholastic principle. In what has become a standard modern reading, this text is taken to express something close to the Boolean equivalence of *All S is P* to *All non-P is non-S* and to be based on a presumption that $S \subseteq P$ iff $-P \cap -S = \emptyset$. The text however equally justifies being interpreted as a statement of Prolcus' equivalence of *All S is P* to *No S is non-P* . If viewed syntactically (setting aside the issue of whether its semantics is Boolean) this reading would "explain" the fact observed by Ł ukasiewicz and others that Aristotle never took advantage of the principle, as understood on the first reading, to reduce other first figure syllogisms to Barbara, for if the second is preferred, the rule does not concern ***E*** statements and could not be used to convert them. See pp. 46-47 in Łukasiewicz, Jan, *Aristotle's Syllogistic*, Second ed, (Oxford: Clarendon Press, 1957). Also see p. 79 in Bochenski, I. M., *A*

no x. It also yields equivalences that appear to be a form of double negation: *y is true of some x* is equivalent to *not not-y is true of some x*, and likewise *y is true of x* is equivalent to *not not-y is true of x*.

Because the canons are clearly rules of logic and are apparently Boolean in nature, they are especially relevant to our discussion and we shall try to reconstruct them with some care. To retain the immediate juxtaposition of the two negation signs, we shall in this section employ the syntactic order of the technical Greek as set out by Aristotle and Ammonius in which the predicate is written first and is "said of" the subject which is written to its right, as in the sentence *y is true of all x*. In later sections on the syllogistic, however, in order to conform to the literature there cited, we shall revert to the more standard syntax (in Greek and English) in which the subject comes first.[2]

Ammonius summarizes the Proclus' rules as follows:[3]

> the consequence is that which is the same in subject and quantity, but is different in both quality and in whether the predicate is simple or transposed .

History of Formal Logic, Second ed, (Notre Dame, Indiana: University of Notre Dame, 1961).

[2] There syllogistic sentences are stated in the abbreviated form in which the letters ***A***,***E***,***I*** and ***O*** are operators joined with terms *x* and *y*, *e.g. y ix true of all x* or equivalently *all x is y* is symbolized ***A****xy*

[3]Pp. 181:30-186:24, esp. 182:3-25 in Ammonius, "*In aristotelis de interpretatione commentarius*," *Commentaria in aristotelem graeca*, Ed. Adolfus Busse, Vol. IV, (Berlin: Gregorius Reimerus, 1895), .. See also Latin trans. Guillaume de Moerbeke, Corpus latinum commentariorum in aristotelem graecorum, G. Verbeke ed, (Louvain: Universitaries de Louvain, 1961).For a partial English translation and discussion, see Allan Bäck, "Ammonius", in Bäck, Allan, *Aristotle on Predication*, (Leiden: Brill, 2000 (forthcoming)).

Here by a *simple* (*haplōs*) *predicate* Ammonius means one that has no preceding negative particle or affix and that is not in a sentence that is as a whole negated. By *transposed* (*ek metatheseōs*) he means the opposite, that either the predicate is preceded by a negative particle or is the predicate of a sentence in which the particle has been "neg-raised" or "fronted" (in the linguist's sense) so that it is the simple predicate of a sentence that is itself negated. For example, in the following sentences both predicates are negated in this sense: *every man is not just* and *not every man is just.* In Aristotle's jargon neg-raising does require the negative particle to move its position. Indeed written in this way both sentences of the example have the same syntax: *not just is true of every man*.

Ammonius explains that transposed or "metathetic" predicates are not meant to include those with a negative affix like the alpha-privative that indicate a "contrary" rather than "contradictory opposite."[4] Their intended interpretation rather seems to be Boolean. A transposed predicate stands for the set-theoretic complement of the simple predicate's extension.

In the example above, for example, the proposition *y is true of all x* of universal quality with two un-negated terms is equivalent to *not y is true of no x* of universal quantity with two negated terms in the

[4] He explains what he means in general by transposed predicate at 161:78 to 162:34.

sense that the sentence's quantity indicated by the subject's modifier is negative and the sentence's predicate is transposed.

The formal reconstruction of the proof theory within the traditional syllogistic is straightforward.

I. Syntax: Let *y* and *x* range over *terms* (predicates and subjects);

Qual over {+,**–** };

Quant over {**U,P,S,I**};

let sentences be any series of expressions of the form:

Qual*y* **Quant Qual***x.*

II. Morphology of Ammonius:

+*y = y;* **–** *y = not y;*

U + *x = all x;*

U **–** *x = no x;*

P + *x = some x;*

P **–** *x = not …. some x;*

S + *x = x;* **S** **–** = *not …. x.* (**S** is for "singular)

I 💻 *x = x;* **I** **–** = *not …. x.* (**I** for "indefinite")

Examples. Sentences with their surface morphology written below them:

+ *y* **U** + *x*	+ *y* **P** + *x*	+ *y* **S/I** + *x*
y is true of all x	*y is true of some x*	*y is true of x*
– *y* **U** + *x*	**–** P **P** + *x*	**–** *y* **S/I** + *x*
not y is true of all x	*not y is true of some x*	*not y is true of x*
+ *y* **U** **–** *x*	+ *y* **P** **–** *x*	+ *y* **S/I** **–** *x*
y is true of no x	*not y is true of some x*	*not y is true of x*
– *y* **U** **–** *x*	**–** *y* **P** **–** *x*	**–** *y* **S/I** **–** *x*
not y is true of no x	*not not y is true of some x*	*not not y is true of x*

III. Proof Theory. Let: *A* be a sentence We adopt the following rules:

(The Canon of Proclus) $\dfrac{(\mathbf{Qual}\,y)\mathbf{QuantQual}\,x}{(-\,\mathbf{Qual}\,y)\mathbf{Quant}-\mathbf{Qual}\,x}$ $\qquad \dfrac{--A}{A}$ $\qquad \dfrac{-+A}{-A}$

Let ┤├ indicate mutual deducibility.

Metatheorems:

+ *y* **U** + *x*	┤├	– *y* **U** – *x*
y is true of all x	┤├	*not y is true of no x*
– *y* **U** + *x*	┤├	+ *y* **U** – *x*
not y is true of all x	┤├	*y is true of no x*
+ *y* **P** + *x*	┤├	– *y* **P** – *x*
y is true of some x	┤├	*not not y is true of some x*
– *y* **P** + *x*	┤├	+ *y* **P** – *x*
not y is true of some x	┤├	*not y is true of some x*
+*y* **S/I** + *x*	┤├	– *y* **S/I** – *x*
y is true of x	┤├	*not not y is true of x*
– *y* **S/I** + *x*	┤├	+ *y* **S/I** – *x*
not y is true of x	┤├	*not y is true of x*

It is intuitively clear (and will be made more precise below) that the canon and the reduction of double negations are valid in the standard interpretation of the syllogistic in terms of a Boolean algebra of non-empty sets, in which predicates stand for non-empty sets, – is set complementation, and 💻 does not alter the interpretation of the predicate.

2. Scalar Logic

Proclus, perhaps more than other Neoplatonists, systematically exploits a feature of natural language that is neglected in classical logic. Ordinary speech typically contains families that consist of a comparative adjective conjoined with a series of one-place adjectives that progressively "name" the points on the total ordering indicated by the comparative. These are called *scalar adjectives*. Examples from English are those associated with *hotter-than* and *happier-than:*

> *is hotter than:* *boiling, hot, warm, tepid, cool, cold, freezing*
>
> *is happier than*: *ecstatic, happy, content, so-so, down, sad, miserable*

In the *Elements of Theology,* for example, Proclus refers to the causal ordering defining the emanations from the One by such comparatives:

> …the higher cause (*aitioterōn*), being the more efficacious (*drastikō teron*), operates sooner upon the participant (for where the same thing is affected by two causes it is affected first by the more powerful (*dunatōteron*); and in the activity of the secondary the higher is co-operative, because all the effects of the secondary are concomitantly generated by the more determinative cause (*aitiōteron*).
>
> ….
>
> All those characters which in the originative causes have higher (*huperteran*) and more universal (*holikōteron*) rank become in the resultant beings, through the irradiation's which proceed from them, a kind of substratum for the gifts of the more specific principles (*merikōteron*). [5]

The various "terms" he uses to describe hypotheses and the various forms, genera and species that occupy the ranks of the causal order function grammatically as monadic scalar adjectives.

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[5] *ET* 66:22-68:2. Such usage of comparatives is frequent. The contexts moreover make it clear that they are meant to refer to the same underlying order. For examples see *ET* 46:19; 58:12; 74:10; 84:14-26; 142:7. In *IP* see 796:14-797:3, M&D 165-166.; 735: 25-29, M&D 110; 892:31-894:34, M&D 253-255; 838:7-14, M&D 211; 1098:3-28, M&D 444-445.

Especially relevant to logic of Proclus is the fact that natural language has several negations that work only in association with scalar families. The first of these presupposes a "direction" to the underlying semantic ordering in which one extreme is designated "positive" and the other "negative." The semantic function of the negation they is to convert a predicate indicating a point in the positive order to one indicating a point on the negative extreme, one roughly as negative relative to a "midpoint" as the other is positive. For example, English uses the prefix *un* in this role, as in ***un****happy,* ***im****polite,* ***in****human*. Algebraically operation may be characterized without the need to presuppose an metric on the order.

The relevant metatheory may be developed within many-valued logic. A (*sentential*) *syntax* is an algebra <Sen,∧,∨,–> such that there is some set A-Sen expressions such that Sen is the closure of A-Sen under the binary operations ∧ and ∨ on expressions and the monadic operation –. A *semantic structure* is an algebra <U, ∧,∨,–> of like character. An *acceptable valuation* relative to the a syntax and semantic structure is any homomorphism from the former to the latter. A *logical matrix* is any <U,D, ∧,∨,–> such that <U, ∧,∨,–> is a semantic structure and D⊆U. D, the set of *designated values*, is used to define entailment: *X entails P* in matrix M (briefly $X \models_M P$) iff for any acceptable valuation *v*, if *v* assigns every expression in *X* a designated value then it assigns a designated value to *P*.

The many-valued theory appropriate for scalar logic are Kleene's *strong connectives*.[6]

Definition. By the (*strong*) 3-*valued Kleene algebra* is meant the structure <{ 0, $\frac{1}{2}$, 1}, max, min, –> such that max and min are respectively the maximum and minimum operations on <{ 0, $\frac{1}{2}$, 1} and – (the *inverse*) is a one-place operation such that –1=0, –0=1 and $-\frac{1}{2}=\frac{1}{2}$.

A *Kleene logical matrix* is any <{ 0, $\frac{1}{2}$, 1}, D_K, max, min, –> such that <{ 0, $\frac{1}{2}$, 1}, max, min, –> is a Kleene algebra and D is a non-empty subset of { 0, $\frac{1}{2}$, 1}.

The Kleene algebra determines the truth-tables for the strong connectives.

	–	∧	0	$\frac{1}{2}$	1	∨	0	$\frac{1}{2}$	1
0	1		0	$\frac{1}{2}$	0		0	$\frac{1}{2}$	1
$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
1	0		0	$\frac{1}{2}$	0		1	$\frac{1}{2}$	1

Depending on whether entailment is taken to preserve "truth" or "non-falsity," the set D_K may be defined as {1} or {1,$\frac{1}{2}$} .

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[6] These opeations are defined by Łukasiewicz in terms of his conditionsal. The semantics for his conditional however assumes a metric on the order sufficient arithmetic operations + and -. Łukasiewicz, Jan, "*On 3-Valued Logic*," in *Jan Ł ukasiewicz, Selected Works*, Ed. I. M. Bochenski, (Amsterdam: North-Holland, 1970). Kleene, S. C., "On a Notation for Ordinal Numbers," *Journal of Symbolic Logic*, 3 (1938), pp. 15-155. The relation of its entailments to classical logic and their syntactic characterizations are well know. See for example Martin, John N., *Elements of Formal Semantics*, (Orlando, FL: Academic Press, 1987).

This three-valued matrix is easily generalized. It is the more abstract characterization is one that embraces scalar orderings. as long the relevant structural conditions are met.

Definition. $< U,\leq,-,e>$ is a *(strong) Kleene* structure iff

1. $<U,\leq>$ is a total ordering
2. $-$ is an operation on U that is antitonic and idempotent
3. $e=-e$

The subset A of U is said to be *closed upwardly* iff for any $x,y\in A$, if $x\in U$ and $x\leq y$, then $y\in A$. Let |A| be the cardinality of A.

Lemma.[7] If $< U,\leq,-,e>$ is a Kleene structure, $3\leq|U|$, and

D_M is an upwardly closed subset of U such *that* either all $x\in D_M$ are such that $e<x$ and $D_K=\{1\}$, or all $x\in D_M$ are such that $e\leq x$ and $D_K=\{1, \frac{1}{2}\}$, then for any X and P, $X\models_M P$ iff $X\models_K P$

Below we shall see that Prolcus makes use of a inverse operation within his causal ordering that meets the structural conditions for the operation – in a Kleene structure. It will also be possible to define in a straightforward way the strong operations $\wedge$ and $\vee$ in that ordering. In this manner it would be possible to extend Proclus' structure to include Kleene $\wedge,\vee$, and – appropriate to the proof theory of Kleene's many-valued logic.[8] Note that to this point we have be able to assume merely that the order has a "midpoint" without needing to posit an

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[7] The proof depends on the fact that there is a onto homomorphism □from $<$ U,max, min,$\tilde{}>$ onto $<\{0, \frac{1}{2}, 1\}$, max, min,$\tilde{}>$ that preserves designation and non-designation ($x\in D_M$ iff □$(x)\in D_K$),

extra assumption that one extreme is designated "positive" and the other negative."

Intensifiers. Scalar adjectives are also associated in natural language with a pair of intensifier "negations," one of which transforms a predicate into one that stands for a "higher" or "more positive" point in the order, and one that transforms it into one that picks out a "lower" or "more negative" point in the order. The former is called by traditional grammarians the *alpha-intensivum.* It is called *hypernegation* by Proclus, and in Pseudo-Dionysius the Areopogite the prefix *hyper* came to be the technical marker of this negation in later Greek Neoplatonism.[9] In English we use this negation, for example, when we say *it is* ***not*** *hot; it 's boiling (**hyper**-hot, **super**-hot).* The second intensifier is called the *alpha-privative* by Aristotle, classical and modern grammarians, and Proclus. We use it in expressions like

> *he does**n't** know what's gong on; he's clue**less***
>
> *he's **sub**par today, **not** his usual self*

In a often cited passage Proclus makes the distinction this way:

> Being , after all, is the classic case of assertion whereas Not-Being is of negation…. So then in every class of Being, assertion in general is superior to negation. But since not-Being has a number of senses, one superior to Being, another which is of the same rank as Being,

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[8] For a proof theoretic characterization of Kleene's 3-valued logic and discussion see Martin, John N., "A Syntactic Characterization of Kleene's Strong Connectives," *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik*, 21 (1975), pp. 181-184.

[9] P. 326. Jespersen, Otto, *The Philosophy of Grammar* (London: Allen and Unwin, 1924).. Horn in his important foundation work scalars uses the grammatical and semantic acceptability of such intensifier to identify scalar adjectives. See Horn also for the linguistic background on scalars employed here. Horn, Laurence R., *A Natural History of Negation* (Chicago: University of Chicago Press, 1989).

> and yet another which is the privation of Being, it is clear, surely that we can postulate also three types of negation, one superior to assertion, another inferior to assertion, and another in some way equally balanced by assertion.[10]

To formulate a predicate from one that describes a point within the level of Being (the level of Ideas), one must use hypernegation (type 1); to form one to speak about levels lower than Being, one must use privative negation (type 3); and to move up and down within the level of being one may use either hyper or privative negation (type 2). These negations in addition conform to principle of Prolcus' logic: affirmation gives birth to negation.[11]

> ….the first mentioned [forms] are more general, while these latter mentioned are more particular. For this reason by eliminating the earlier ones, he eliminates those that follow them in the hypotheses.

More generally:[12]

> If, then, the negations generate the affirmations, it is plain that the first negations generate the first and the second the second.

Algebraically this rule expresses the restriction that hyper and privative negation are order-preserving. It is a straightforward matter to define the syntax and semantics for a language of scalars that, as such, fits Proclus usage. The semantics presupposes that a "midpoint" for the background order is distinguished.

Definitions

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[10] *IP* 1072:28-1073:8, M&D 426

[11] *IP* 1087:2-6, M&D 435.

A *symmetric scalar syntax* is any $<\{P_n,\ldots,P_0,\ldots,P_{-n}\},\text{Pred},\sim,\neg,->$ in which for some *Y*,

a. $\{P_1\ldots,P_n\}$ is a set of expressions, the *atomic monadic predicates*;

b. $\sim$, $\neg$, and $-$ are one-place operators; and

c. Pred is the closure of atomic predicates under $\sim$,$\neg$, and $-$.

A *symmetric scalar structure* is any $<\text{U}, \leq,-,\sim,\neg, e>$ such that

a. $\leq$ is a total order on U

b. $\sim$ and $\neg$ are isotonic binary operations on $<\text{U},\leq>$

c. for any $x\in\text{U}$, $\neg x\leq x\leq \sim x$

d. $-$ is an antitonic idempotent binary operation on U

e. $e=-e$

Relative to a symmetric scalar syntax $<\{P_n,\ldots,P_0,\ldots,P_{-n}\},\text{Pred},\sim,\neg,->$ and structure $<\text{U}, \leq,-,\sim,\neg, e>$, an *interpretation* is any homomorphism R from $<\text{Pred},-,\sim,\neg,P_0>$ into $<\text{U},-,\sim,\neg,e>$ such that

a. for any *i* and *j*, if $i\leq j$, then $\text{R}(P_j)\leq\text{R}(P_i)$

b. for any *i*, $\text{R}(P_i)\in\text{U}$,

i. $\text{R}(P_i)\leq \text{R}(\sim P_i)$

ii. $\text{R}(\neg P_i)\leq \text{R}(P_i)$

c. for any $i\in\text{U}$, and $-\text{R}(P_i)\leq \text{R}(P_i)$,

Theorem. If $<\text{U}, \leq,-,\sim,\neg,e>$ is a symmetric scalar structure, then $<\text{U}, \leq,-,e>$ is a Kleene structure.

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[12] *IP* 1099:32-35.

3. The Syllogistic

The Aristotelian Syllogistic. For comparison we now state the standard natural deduction reconstruction of the syllogistic in the manner of Smiley and Corcoran modeled on Aristotle's "reductions" of the *Prior Analytics* [13] Deductions with any finite number of premises are allowed, These will include traditional immediate inferences, syllogisms, and many premised syllogistic arguments traditionally represented by chains of syllogisms. The rules of the natural deduction system are that "reduces" the valid moods to Barbara and Celarent using various immediate inferences and reduction to the impossible. The semantics defines the truth-conditions of the traditional four forms in terms of the standard lattice operations $\wedge$, $\vee$ and $-$. Though it is not necessary to assume that the operations are Boolean, it is necessary to assume that there is a least element 0 that no term stands for and from which the order $\leq$ receives a "positive/negative" direction.

The theory posits a syllogistic syntax in which sentences are made up by attaching one of four sentence operators ***A, E, I*** and ***O,*** to pairs of expressions drawn form a primitive set of terms.

Definition. A *syllogistic syntax* is any $<\{P_1,\ldots,P_n,\ldots\}$,Sen,***A***,***E***,***I***,***O***$>$ such that

1. $\{P_1,\ldots,P_n,\ldots\}$ (possibly denumerably infinite), called the set of *terms,* is a set of expressions,

[13] Corcoran, John, "Completeness of an Ancient Logic," *Journal of Symbolic Logic*, 37 (1972), pp. 696-702. Smiley, Timothy, "What is a Syllogism?," *Journal of Philosophical Logic*, 2 (1973), pp. 136-154. The metatheory here is developed in

2. ***A***,***E***,***I***, and ***O*** are two-place syntactic operators defined on pairs of terms that yield the concatenation of the operator and the ordered terms,

3. the union of their ranges is Sen, the set of *sentences*.

Let *x*, *y* and *z* range Terms. (We conform to English syntax below placing the "subject" to the left of the "predicate".) The following definitions are adopted for the introduction of an eliminative contradictory sentential negation operator **N:** $\mathbf{N}\boldsymbol{A}xy =_{def} \boldsymbol{O}xy$, $\mathbf{N}\boldsymbol{E}xy =_{def} \boldsymbol{I}xy$, $\mathbf{N}\boldsymbol{I}xy =_{def} \boldsymbol{E}xy$, $\mathbf{N}\boldsymbol{O}xy =_{def} \boldsymbol{A}xy$.

Definitions

A *basic deduction* is any $X \vdash P$ such that X is finite and $P \in X$.

The set of acceptable deduction rules are:

Conversion1: $\dfrac{X \vdash \boldsymbol{E}xy}{X \vdash \boldsymbol{E}yx}$ Conversion2: $\dfrac{X \vdash \boldsymbol{A}xy}{X \vdash \boldsymbol{I}xy}$ Reductio: $\dfrac{X \vdash A \quad Y \vdash \mathbf{N}A}{X \cup Y - \{B\} \vdash \mathbf{N}B}$

Barbara: $\dfrac{X \vdash \boldsymbol{A}zy \quad Y \vdash \boldsymbol{A}xz}{X, Y \vdash \boldsymbol{A}xy}$ Celrarent: $\dfrac{X \vdash \boldsymbol{E}zy \quad Y \vdash \boldsymbol{A}xz}{X, Y \vdash \boldsymbol{E}xy}$

The set $\vdash_{syl}$ of *provable deductions* is the least set including the basic deductions and closed under the rules. ($X \vdash_{syl} A$ means $X \vdash A$ is provable).

A *syllogistic structure* is any <U,≤,∧,0> such that

a. <U,≤> is a partially ordered structure with least element 0;

b. <U,∧> is the meet semi-lattice determined by <U,≤>.

Martin, John N., “Aristotle's Natural Deduction Reconsidered,” *History and Philosophy of Logic*, 18 (1997), pp. 1-15.

2. A *syllogistic interpretation* relative to <U,≤,∧,0> is any function R of Syn mapping Terms∪Sen to U∪{T,F} such that:

a. if x∈Terms, R(x)∈ U and R(x)≠0,

b. if A∈Sen, then

i. if A is some ***A****xy*, then R(A)=T iff R(x)≤R(y),

ii. if A is some ***E****xy*, then R(A)=T iff R(x) ∧R(y)=0,

iii. if A is some ***I****xy*, then R(A)=T iff R(x)∧R(y)≠0,

iv. if A is some ***O****xy*, then R(A)=T iff not(R(x)≤R(y)).

An argument X to A is *(syllogistically) valid* (briefly $X\models A$) iff for any syllogistic interpretation R of a syllogistic structure for the syllogistic syntax, if for all B∈X, R(B)=T, then R(A)=T.

The natural deduction proof and semantics is an abstraction from Corcoran and Smiley and permits lattices as acceptable structures which are not Boolean algebras of sets.[14] Because of its increased generality it is a more accurate characterization the semantic structures delimited by the natural deduction rules that serve as the Corcoran-Smiley reconstruction of the logic of the *Prior Analytics.*

Theorem. $X\vdash_{syl}A$ iff $X\models A$.

<u>The Neoplatonic Scalar Syllogistic</u>. Though the syllogistic syntax just defined is standard and modeled on Aristotle's, it does not posses a predicate negation appropriate for representing the canons attributed to Prolcus by Ammonius.

It would be a simple matter to sketch a non-Neoplatonic syntax and semantics suitable to the task, one that conformed to the structure of classical Boolean algebra. Let us call the relevant predicate operator *not.* In the relevant semantics a predicate *x* would stand for a non-empty set in a Boolean structure and its negation *not x* would stand for the set theoretic complement of the extension of *x.* As Corcoran and Smiley demonstrate, the standard proof theory is sound and complete for semantics in which interpretations are restricted to Boolean structures of non-empty sets. Moreover it is a simple matter to check that Proclus' canons construed in terms of *not* and the sentence operator **N** would be also be valid in interpretations restricted to these Boolean structures. However, the canons in this sense will not be pursued here for two reasons. We shall see that neither are the canons valid in the more abstract semantics given above nor is a Boolean semantics appropriate for Proclus' version of the syllogistic.

The syllogistic may be extended to make it scalar. The terms will then stand for points in a scalar ordering and operations ~,¬, and – will be introduced as scalar negations. The ***A*** forms will then express causal order. The semantic structure combines symmetric scalar and a scalar syllogistic structures. Since every term stands for a non-minimal element (in both Aristotle and Proclus), the privative operation ¬ does not have as a value the syllogistic least element 0. Let R|A be stand for the restriction of the relation ∪ to A. The portion of the universe over which syllogistic terms are interpreted is restricted so as not to

[14] The proof is to be found in Martin(1997).

include 0. Since 0 is not the referent of any term, the semantic operation – is defined for it.

Definitions. A *symmetric scalar syllogistic syntax* is any <{P_n,…,P_0,…,P_{-n}},Terms, Sen, ***A***,***E***,***I***,***O***,~,¬,–> such that

1. <{P_n,…,P_0,…,P_{-n}},Terms,~,¬,–> is a symmetric scalar syntax,
2. <Terms,Sen,***A***,***E***,***I***,***O***> is a syllogistic syntax.

A *basic deduction* is any $X\vdash P$ such that $P\in X$ or $X\vdash P$ is of one of the forms:

$\varnothing\vdash \mathbf{I}xy$, $\varnothing\vdash \mathbf{A}\neg xx$, or $\varnothing\vdash \mathbf{A}x{\sim}x$ $\varnothing\vdash \mathbf{A}{-}xx$, $\varnothing\vdash \mathbf{A}{-}{-}xx$,

$\varnothing\vdash \mathbf{A}x{-}{-}x$, $\varnothing\vdash \mathbf{A}P_0 {-}P_0$, and $\varnothing\vdash \mathbf{A}{-}P_0 P_0$.

The set of *acceptable deduction rules* has as its elements: Conversion1, Conversion2, Reductio Barbara, Celarent and the following rules:

$$\frac{X\vdash \mathbf{A}xy}{X\vdash \mathbf{A}\neg x\neg y}\qquad \frac{X\vdash \mathbf{A}\neg x\neg y}{X\vdash \mathbf{A}xy}\qquad \frac{X\vdash \mathbf{A}xy}{X\vdash \mathbf{A}{\sim}x{\sim}y}\qquad \frac{X\vdash \mathbf{A}{\sim}x{\sim}y}{X\vdash \mathbf{A}xy}\qquad \frac{X\vdash \mathbf{O}xy}{X\vdash \mathbf{A}yx}$$

$$\frac{X\vdash \mathbf{A}xy}{X\vdash \mathbf{A}{-}y{-}x}\qquad \frac{X\vdash \mathbf{A}{-}x{-}y}{X\vdash \mathbf{A}yx}$$

The set $\vdash_{syl+}$ of *provable deductions* is the least set including the (new) set of basic deductions and closed under the (new) set of rules. ($X\vdash_{syl+}A$ means $X\vdash A$ is provable in the wider system).

A *symmetric scalar syllogistic structure* is a structure <U,≤,∧–,~,¬,*e*,0> such that

a. <U–{0}, ≤|U–{0},~,¬,–|U–{0},*e*> is a symmetric scalar structure,

b. <U,≤,∧,0> is a syllogistic structure,

c. –0 is defined and in U.

A *symmetric scalar syllogistic interpretation* relative to

<{P_n,…,P_0,…,P_{-n}},Terms, Sen, ***A***,***E***,***I***,***O***,~,¬,–> and <U,≤,∧–,~,¬,*e*,0>

is any function R such that

a. R is a symmetric scalar interpretation relative to

<{P_n,…,P_0,…,P_{-n}},Terms,~,¬,–> and <U–{0}, ≤|U–{0},~,¬,

–|U–{0},*e*> , and

b. R is a syllogsitic interpretation relative to <Terms,Sen,***A***,***E***,***I***,***O***>

and <U,≤,∧,0> .

An argument *X* to *A* is *valid* (briefly $X \models A$) relative to a symmetric scalar syllogistic syntax and structure iff for any symmetric scalar syllogistic interpretation R , if for all $B \in X$, R(*B*)=T, then R(*A*)=T.

Theorem

1. If <U,≤,∧–,~,¬,*e*,0>is a symmetric scalar syllogistic structure , then –0 is the unique ≤ supremum (call it 1) in U.
2. **Ineffability**. If R is symmetric scalar syllogistic interpretation R, then neither R(*P*)=1 nor R(*P*)=0.
3. For any symmetric scalar syllogistic structure <U,≤,∧–,~,¬,*e*,0> relative to a scalar syllogistic syntax, <U, ≤,–,*e*> is a Kleene structure and <U, ≤,–,~,¬, *e*> is a symmetric scalar structure.

4. **Soundness and Completeness.**[15] $X \vdash_{syl+} A$ iff $X \models A$.

4. The Neoplatonic Canons

One of the more curious consequences of the linear syllogsitic is that Proclus cannot be understood as accepting the canons attributed to him by Ammonius as these were interpreted earlier in the paper in terms of Boolean set complementation. The rules as there formulated are invalid in the scalar syllogistic. They cannot hold because the ***A*** statements are non-trivial, whereas the purported equivalents that are stated as ***E*** and ***I*** statements are trivially true and false respectively.

Theorem. The deductions set out in the canons of Proclus, in the earlier morphology of Ammonius, are invalid in the scalar syllogistic.

Indeed if Proclus does think causation and predication are one, and that they describe a causal line, it would be odd for him to defend the canons under the earlier formulation. After all, the atomic facts he wants to express are all captured by non-trivial ***A*** and ***O*** propositions, whereas the alleged ***E*** and ***I*** equivalents are in general vacuous.

How then can Proclus both adopt the syllogistic and the canons attributed to him by Ammonius? The answer is to be found in an alternative reading of intended sense of negation. If a term's "quality" is not defined in terms of the contradictory opposite negation of ***E*** and ***O*** propositions, but is rather defined in terms of the scalar negations $\sim$ and $\neg$, the canons become valid in the scalar

[15] The proof is an extension of that of Martin (1997) and is forthcoming. See "Logic of Proclus," abstract, Association for Symbolic Logic, 2000 Annual Meeting, June 3-7, University of Illlinoi at

syllogsitic. As Ammonius' rule would have it, the following equivalents reverse a term's the positive and negative "quality" while keeping the proposition's quantity constant:

Theorem. The Canons of Proclus. In the symmetric scalar syllogistic,

$\boldsymbol{A}xy$	$\dashv\vDash$	$\boldsymbol{A}{\sim}x{\sim}y$	$\dashv\vDash$	$\boldsymbol{A}\neg x\neg y$
$\boldsymbol{E}xy$	$\dashv\vDash$	$\boldsymbol{E}{\sim}x{\sim}y$	$\dashv\vDash$	$\boldsymbol{E}\neg x\neg y$
$\boldsymbol{I}xy$	$\dashv\vDash$	$\boldsymbol{I}{\sim}x{\sim}y$	$\dashv\vDash$	$\boldsymbol{I}\neg x\neg y$
$\boldsymbol{O}xy$	$\dashv\vDash$	$\boldsymbol{O}{\sim}x{\sim}y$	$\dashv\vDash$	$\boldsymbol{O}\neg x\neg y$

Construed in this Neoplatonic manner, moreover, the canons are not simply convenient conversion rules.[16] They describe a logical relation that is fundamental indeed. They express the isometric property of hyper and privative negation, the law that underlies negative knowledge of higher hypotheses.

There is explanatory importance in the fact that the canons have valid versions in the syllogistic as interpreted both in terms of non-empty sets (though not in terms of the more abstract lattices) and in terms of "properties" in a scalar order. It allows us to explain why Proclus defends the canons, as Ammonius tells us he does, and why Ammonius presumes a set theoretic interpretation in commenting on them. Prolcus is mainly concerned with Neoplatonic metaphysics which has a different structure from that of sets. Hence he uses the scalar negations. Ammonius is explicating Aristotle's logic as a self-contained corpus considered in isolation from Neoplatonic metaphysics. He interprets it in the manner of Aristotle for whom the genera and species picked

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[16] It should be remarked that since the law of double negation fails for both hyper and privative negations, these equivalences do no hold for any uniform replacement of a term by its negation and of a negation by its un-negated form. For example, let $x={\sim}y$. Then $\boldsymbol{A}x{\sim}{\sim}y$ but not $\boldsymbol{A}{\sim}x{\sim}y$.

out by terms have a structure which reflects that of their non-empty set theoretic extensions.

In the commentary tradition it is often difficult to tell what the commentator thinks about the views he is discussing. Ammonius is discussing Aristotle's logic and does not tell us whether or how he would reconcile its radically different ontology with that of his teacher. It may even be the case that Prolcus' hypothetical lectures on Aristotle's syllogistic are closely reflected in what his pupil writes. If so, in these lectures Proclus too would be adopting the commentator's stance in which it is common to reserve judgment on the truth of the view under discussion. On the other hand, it would be surprising if teacher and pupil clearly understood the systematic differences between Prolcus' scalar syllogistic and Aristotle's syllogistic interpreted in terms of non-empty sets. They simply did not posses the analytical tools needed for such a comparison. Ammonius did explain Aristotle's logic at length, and presumably he derived some of this theoretical knowledge from his teacher. Clearly, the texts indicate that both could work well and consistently within the framework they are discussing. Nevertheless, neither gives evidence of the using the metatheoretic concepts necessary for comparing the effect on logical inference of the two interpretive frameworks.

Bibliography

Ammonius, *Commentarie sur le Peri hermeneias d'Aristotle*, Trans. Guillaume de Moerbeke, Corpus latinum commentariorum in aristotelem graecorum, G. Verbeke ed, (Louvain: Universitaries de Louvain, 1961).

Ammonius, "*In aristotelis de interpretatione commentarius*," Ed. Adolfus Busse, in *Commentaria in aristotelem graeca*, Vol. IV, (Berlin: Gregorius Reimerus, 1895), .

Bäck, Allan, *Aristotle on Predication* (Leiden: Brill, 2000 (forthcoming)).

Bochenski, I. M., *A History of Formal Logic*, Second ed. (Notre Dame, Indiana: University of Notre Dame, 1961).

Corcoran, John, "Completeness of an Ancient Logic," *Journal of Symbolic Logic*, 37 (1972), pp. 696-702.

Horn, Laurence R., *A Natural History of Negation* (Chicago: University of Chicago Press, 1989).

Jespersen, Otto, *The Philosophy of Grammar* (London: Allen and Unwin, 1924).

Kleene, S. C., "On a Notation for Ordinal Numbers," *Journal of Symbolic Logic*, 3 (1938), pp. 15-155.

Lukasiewicz, Jan, *Aristotle's Syllogistic*, Second ed. (Oxford: Clarendon Press, 1957).

Lukasiewicz, Jan, "*On 3-Valued Logic*," *Jan Lukasiewicz, Selected Works*, Ed. I. M. Bochenski (Amsterdam: North-Holland, 1970), .

Martin, John N., “Aristotle's Natural Deduction Reconsidered,” *History and Philosophy of Logic*, 18 (1997), pp. 1-15.

Martin, John N., *Elements of Formal Semantics* (Orlando, FL: Academic Press, 1987).

Martin, John N., “A Syntactic Characterization of Kleene's Strong Connectives,” *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik*, 21 (1975), pp. 181-184.

Proclus, “*Procli Commentarium In Platonis Parmenidem*,” Ed. and Trans. Victor Cousin, *Procli philosophi platonici operas inedita* (Paris: Augustus Durand, 1864), pp. 603-1313.

Proclus, *Proclus' Commentary on Plato's Parmenides*, Trans. Glenn R. Morrow, John M. Dillon (Princeton, NJ: Princeton University of Press, 1987).

Proclus, *Proclus: The Elements of Theology*, Trans. E. R. Dodds, Second ed. (Oxford: Clarendon Press, 1963).

Proclus, *Théologie Platonicienne*, Trans. H.D. Saffrey and L.G. Westerink, Vol. I-VI (Paris: Les Belle Lettres, 1968-1997).

Smiley, Timothy, “What is a Syllogism?,” *Journal of Philosophical Logic*, 2 (1973), pp. 136-154.