Regularized Kirchhoff Rod model in a Brinkman fluid
Nguyenho Ho, Karin Leiderman, and Sarah Olson

Abstract: We investigate 3-dimensional motility of sperm in fluids with a sparse network of stationary obstacles or fibers. The Brinkman equation is used to model the average fluid flow where a resistance term is inversely proportional to the permeability of the porous medium and represents the effect due to the presence of the fibers. The sperm flagellum is idealized as a Kirchhoff rod that can exhibit lateral or spiral waves. To solve for the local fluid velocity and angular velocity, we use the method of regularized Brinkmanlets and extend it to the case for a Kirchhoff rod that is discretized as point forces and torques. The numerical method is validated by comparing to asymptotic swimming speeds derived for a cylinder of infinite length that is propagating lateral or spiral waves in a Brinkman fluid. Similar to the asymptotics, we observe that in the case of small amplitude, swimming speed is enhanced as the permeability in the fluid is decreased. For larger amplitudes, the simulations show a non monotonic change in swimming speed as the fluid permeability is increased. This is due to the emergent amplitude and wavelengths; as the permeability is decreased, the emergent amplitude of the swimmer has a tendency to decrease due to the extra resistance of the increased stationary fibers.