Cognitive Effects as Distribution Rescaling

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A reanalysis of lexical decision response time distributions from Holden (2002) reveals a self-similar rescaling of response time distributions in the effect of inconsistency in word’s pronunciation-to-spelling relationships. Scaling arises in a wide array of neurophysiological and behavioral systems. A general basis for the emergence of these patterns in biological systems remains under debate. However, biological processes that themselves unfold on timescales ranging from fast to slow often express fractal scaling. One possible basis for self-similar fractal distributions arising for cognitive activity is the fundamental mismatch between relative biological time and absolute clock time.

Guy Van Orden and ecological psychologists shared a sentiment that “there is no cognition in a vacuum.” In fact, Guy’s critiques of the subtractive logics of double dissociation and functional neuroimaging are neatly summarized as demonstrations that cognition in the absolute vacuum of space is in some sense required for those enterprises to succeed. That is, the pieces of a conventional cognitive system must operate by the same rules regardless of their immediate context. By the early 1990s, if not sooner, Guy discarded all modular logics of cognition.

If not a modular system, then what kind of system did scientists confront? Guy explored various ideas with his colleagues and students. Initially no clear paths to an answer presented themselves. The dynamic tools that unlocked problems
in motor control did not, at first, appear to straightforwardly apply to standard laboratory-based cognitive tasks. In the end, concepts from fractal geometry helped span the empirical gap, allowing Guy and his clan of collaborators to finally enfold dynamical principles into cognitive narratives. With this article, I hope to add one more truss to bridge the gap.

Guy first noted how, in English, the pattern of statistical relations between spelling patterns and pronunciations could be conceived as a nested and self-similar pattern (Van Orden & Goldinger, 1994; Van Orden, Pennington, & Stone, 2001). Moreover, Greg Stone’s feedback consistency effect established that the route to resolving ambiguity in those relations entailed bidirectional and iterative flow that commonly governs the dynamics of fractal systems (Stone, Vanhoy, & Van Orden, 1997).

This article presents a reanalysis of lexical decision response times from Holden (2002) to illustrate yet another potentially important role for fractal geometry in understanding cognitive activity. The original article illustrated how increasingly ambiguous pronunciation-spelling relations affected the shape of pronunciation and response time distributions. The shape changes impacted the distribution’s relative variability and skew, and those changes are what drove the observed mean differences associated with the manipulations. With hindsight provided by the cocktail distribution (Holden & Rajaraman, 2012), I now realize that the shape changes resulting from cognitive manipulations are often strikingly consistent with a self-similar, proportional rescaling of a distribution’s shape, relative to a baseline distribution.

In what follows, I first briefly characterize the nested sources of ambiguity in the relations among English spelling patterns and pronunciations. I then investigate the rescaling hypothesis in the context of the effect of feedback consistency. Finally, my General Discussion is focused on potential neurophysiological implications of distribution rescaling in cognitive activity.

NESTED SOURCES OF AMBIGUITY

In English, when the grapheme B appears at the beginning of a word, it is always pronounced /b/. Likewise, the phoneme /b/ is always spelled B. It is an invariant and symmetric spelling-pronunciation relation. By contrast, spelling-pronunciation relations among vowels and phonemes are more variable. For instance, the vowel pronunciations of the spelling pattern _ u _ _ are consistent with those entailed in the words duck, burn, and dude. Likewise, the phoneme /u/ could be alternately spelled _ u _ _, _ o _ _, and _ o e _ as in duck, monk, and does. The ambiguity among these bidirectional relationships is normally resolved by the additional contextual constraints supplied by additional letters.
More letters provide more detailed sources of constraint with respect to the potential to transform an ambiguous spelling pronunciation-relation into a fully determined and invariant relation. For instance, although the vowel pronunciation of _u_ _ _ is underdetermined, every English word that entails the _uck spelling body is pronounced /uk/ to rhyme with duck. Likewise, all English words that entail the /uk/ pronunciation rime rely on the _uck spelling-body. Thus, the _uck ⇔ /uk/ spelling-body pronunciation-rime relation is invariant. Most English body-rime relations are ambiguous, however. For instance, _ave can be pronounced as in have or as in gave; it is body-rime ambiguous. Similarly, in English the vowel pronunciation rime /urn/ can be legitimately spelled as in turn or as in fern. Such relations are the mirror image of body-rime ambiguity and are said to be rime-body ambiguous.

For most words, vowel-pronunciation and spelling-pattern ambiguity at the level of spelling bodies and pronunciation rimes is fully resolved with the addition of the constraint added by their whole-word spelling pattern. Words such as gave and turn have only one legitimate English spelling pattern and only one legitimate English pronunciation. Nevertheless, there are at least two exception word classes. The homograph spelling pattern lead maps into two legitimate pronunciations with different meanings. One pronunciation rhymes with bead, the other with head. Similarly, the homophone pronunciation /hare/ maps into two legitimate spelling patterns—hair and hare—two separate words with distinct meanings.

Various words entail varying degrees of these three nested levels of ambiguity. The present reanalysis of Holden (2002), Experiments 3 and 4, examines lexical decision performance to all three “scales” of that nested pattern in terms of the ambiguity entailed by rime-body and whole-word pronunciation-to-spelling relations. That is, lexical decision performance to invariant rime-body words is contrasted to rime-body ambiguous items, which is, in turn, contrasted to homophones, or whole-word spelling ambiguous items.

**DISTRIBUTION SCALING**

The recently formalized cocktail model of pronunciation and response time distributions allows for rescaling tests. The cocktail model represents response time distributions as a mixture of samples originating from ideal lognormal and inverse power law distributions (Holden & Rajaraman, 2012; Holden, Van Orden, & Turvey, 2009). When depicted on a logarithmic scale, both ideal distributions are scale invariant.

Shifting the lognormal mean results in a proportional rescaling of the distribution’s variance. Thus, multiplying a lognormal by a constant (> 1) in the linear
domain has the same effect as adding a constant in the logarithmic domain. On the linear scale, both the mean and variability increase. On the log scale, as the mean increases the variability remains constant. In other words, on the logarithmic scale a lognormal behaves as a Gaussian on a linear scale. Inverse power-law distributions are similarly scale invariant with respect to their location parameter, or onset threshold. Effectively, location shifts of the cocktail mixture distribution, in the logarithmic domain, indicate a proportional rescaling of the distribution.

As such, the cocktail distribution is useful in tests for patterns of change that are consistent with scale invariance. Given the cocktail model, an ideal proportional rescaling is indicated by a relative broadening in the distribution’s shape that selectively influences the model’s location parameters ($\Omega_{\text{LN}}$ and $\Omega_{\text{PL}}$). In the context of an empirical effect, rescaling is indicated if (a) the cocktail distribution reasonably describes the empirical distribution and (b) the cognitive manipulation exclusively shifts the location parameters of the cocktail distribution but leaves the values of the other parameters intact (e.g., $\sigma$, $\alpha$, $\rho_{\text{FLN}}$, $\rho_{\text{BLN}}$, and $\rho_{\text{PL}}$). These additional parameters reference different aspects of the distribution’s relative variability (for details, see Holden & Rajaraman, 2012; van Rooij, Nash, Rajaraman, & Holden, 2013). Figure 1 illustrates three parameterizations of the cocktail distribution and depicts an idealized pattern of scaling in a distribution. Rescaling in natural systems is likely more variable and idiosyncratic than ideal rescaling. Statistical resampling or bootstrapping techniques are used in conjunction with the cocktail model to determine the extent that a given manipulation corresponds to a rescaling pattern.

Next, I fit the cocktail model to response time distributions that manipulated relative ambiguity in the feedback direction, from pronunciations to spelling patterns. I then use bootstrapping techniques to test the degree to which the manipulations affected performance in a manner consistent with distribution rescaling.

**METHOD**

The full details of the experimental design, descriptions of the laboratory procedures, and the considerations that guided item selection and yoking appear in Holden (2002).

**Participants**

Seventy-four introductory psychology students participated in exchange for course credit.
This plot illustrates an idealized distribution rescaling pattern as expressed by multiplying the baseline distribution’s two location parameters by the same constant value (dashed line). The dotted line depicts the resulting first rescaling distribution. For the second rescaling distribution (solid line), the location parameters of the first rescaling distribution were multiplied by a constant that was twice the value (on a log scale) of that used for the first rescaling. The rescaling operation yields correlated increases in the distribution’s linear mean and standard deviation, as is the case for a pure lognormal distribution. If the shape change deviates from rescaling with an increase in power law proportion ($\rho_{PL}$) or a decrease in the scaling exponent ($\alpha$) then the distribution’s skew tends to increase as well.

Stimuli

The key experimental stimuli for the first study were 20 yoked pairs of body-rime invariant and rime-body ambiguous targets. The second experiment used 20 different yoked pairs of rime-body ambiguous and whole-word spelling ambiguous items. In both cases, each yoked item pair was matched for other factors, such as length and word frequency, that are known to affect response time. Complete item lists appear in the appendices of Holden (2002).

Procedure

A standard lexical decision task presented individual letter strings on each of 356 experimental trials. Participants pressed a “yes” key if the presented item
was a word (e.g., goat) and a “no” key otherwise. Nonexperimental filler word trials were added so that half the trials were word trials and the other half were catch trials that presented pronounceable nonword letter strings (e.g., glurp). The key dependent measures were response time and response accuracy.

RESULTS

Only observations for which a given participant responded correctly to both yoked items were included in the following analyses. In addition, both observations had to fall within a 200–2,500 ms response time interval. The upper plot in Figure 2 depicts two variable-width kernel smoothed probability density functions derived from the Experiment 3 response times from Holden (2002) in black. The dashed black line depicts the distribution of correct response times to the baseline rime-body invariant items. Likewise, the solid black line depicts
the distribution of correct response times to the rime-body ambiguous items. Both distributions display signs of bimodality. The bimodality likely arose as a consequence of the range of the yoked item pair’s word frequency counts. A few yoked pairs had moderately high frequency counts (e.g., \(\sim5–11\) per million); others had very low frequency counts (\(\sim1–2\) per million; Kučera, & Francis, 1967).

The dashed and solid white lines in the upper plot of Figure 2 depict cocktail model fits to the rime-body invariant and ambiguous distributions. In both cases the unimodal cocktail model accommodated the apparent bimodality of the empirical distributions by bracketing its single mode between each of the empirical distribution’s two modes. Statistical tests indicated that the empirical bimodality was not sufficient to exclude the unimodal cocktail distribution as a plausible description of the empirical distributions (Invariant \(p = .50\) & Ambiguous \(p = .59\)). The \(p\) values were estimated using a conservative three-step Monte Carlo goodness-of-fit test described by Clauset, Shalizi, and Newman (2009).

Bootstrap resampling on the empirical rime-body invariant and ambiguous response time distributions tested for changes in each of five key free parameters of the cocktail model (\(\Omega_{LN}, \sigma, \alpha, \rho_{PL}\), and \(\Omega_{PL}\)). Both distributions were repeatedly and randomly resampled, with replacement and fit with the cocktail model. The bootstrap fitting procedure was repeated 200 times on both the baseline invariant and ambiguous rime-body distributions. Each time the parameters resulting in a “best fit” were retained.

Again, an ideal proportional rescaling is indicated when only the lognormal and inverse power law location parameters are affected by the manipulation. A series of \(Z\) tests, using standard errors derived from the bootstrap normal approximation technique (Mooney & Duval, 1993) indicated that both the location parameters were reliably influenced by the manipulation \(Z = 4.20, p < .05\) (Invariant \(\Omega_{LN} \ M = 6.37, SD = .013\), Ambiguous \(\Omega_{LN} \ M = 6.46, SD = .017\), in Natural log units) and \(Z = 2.32, p < .05\) (Invariant \(\Omega_{PL} \ M = 6.47, SD = .027\), Ambiguous \(\Omega_{PL} \ M = 6.58, SD = .035\), in Natural log units), respectively. However, the rime-body ambiguous distribution’s power law scaling exponent was reliably smaller than the rime-body invariant scaling exponent, \(Z = -2.29, p < .05\) (Invariant \(\alpha \ M = 5.4, SD = .26\), Ambiguous \(\alpha \ M = 4.67, SD = .19\)). The smaller scaling exponent indicates that the ambiguous items gave rise to a relatively more stretched, slow tail than the invariant distribution. Overall, the pattern of change in the distribution associated with the manipulation is consistent with both a rescaling and an increase in power law behavior. Thus, the shape change amounted to pattern of stretching that fell slightly beyond that of a simple proportional rescaling.

Similarly, the dashed black line in the lower plot of Figure 2 depicts the distribution of response times to the baseline rime-body ambiguous items of
Experiment 4 of Holden (2002). The solid black line depicts the response time distribution to the yoked homophones that entail additional whole-word pronunciation scale ambiguity. The modest bimodality was again due to the range of word frequency counts across the various yoked item pairs. Once again, the “effect” of the additional ambiguity entailed by the multiple potential ways to spell the pronunciation of whole words, such as *blew*, approximated a rescaling.

As before, the dashed and solid white lines in the lower plot of Figure 2 depict cocktail model fits to the rime-body ambiguous and homophone distributions, respectively. Statistical tests indicated that the apparent bimodality was not sufficient to exclude the unimodal cocktail distribution as a plausible description of the empirical distributions (Invariant $p = .94$ & Ambiguous $p = .59$). Again, the $p$ values were estimated using a three-step Monte Carlo goodness-of-fit test described by Clauset et al. (2009).

As before, a bootstrapping analysis indicated the manipulation approximated a proportional rescaling relative to each baseline distribution. Z tests indicated that both location parameters were reliably influenced by the manipulation $Z = 3.50, p < .05$ (Ambiguous $\Omega_{LN} M = 6.39, SD = .013$, Homophone $\Omega_{LN} M = 6.47, SD = .02$, in Natural log units) and $Z = 2.27, p < .05$ (Ambiguous $\Omega_{PL} M = 6.49, SD = .025$, Homophone $\Omega_{PL} M = 6.59, SD = .038$, in Natural log units), respectively. In this case, the exception to an ideal rescaling pattern was indicated by a reliable increase in the lognormal standard deviation parameter across conditions $Z = 1.99, p < .05$ (Ambiguous $\sigma M = .16, SD = .01$, Homophone $\sigma M = .19, SD = .012$). The increase in the standard deviation parameter suggests the presence of slightly more variability in the modal region of the homophone response time distribution than is expected in the case of an ideal rescaling.

Overall, both manipulations induced shape changes that closely but imperfectly approximated an ideal rescaling pattern. In both cases the observed shape changes entailed variability increases that were slightly in excess of the pattern of change predicted by an ideal proportional rescaling. This outcome is not surprising in the case of a statistical pattern. Deterministic mathematical fractals obey precise scaling patterns. Scaling observed in statistical fractals tends to be more idiosyncratic.

In biological systems, governing dynamics are influenced by extrinsic and intrinsic sources of noise and flux, so there is a potential for additional and variable sources of uncertainty to influence observables. For instance, both data sets were aggregated across many individual participants. The yoked-item, within-participants experimental design was aimed at controlling, as much as possible, for individual differences. However, each individual participant entails a unique history of interaction with his or her linguistic environment. Different items are likely encountered at variable rates across different individuals, but counts of word frequencies offer a single rate estimate for all participants.
Likewise, relative subword spelling and pronunciation relations are themselves computed from the same whole-word frequency counts.

GENERAL DISCUSSION

Accounts that posit word recognition as mediated by separate mechanisms or lexical routes most naturally predict simple location shifts in response time distributions. This helps explain the intense focus on mean contrasts in the word recognition literature. However, the feedback consistency manipulation did not simply shift the location of the ambiguous-item distribution to a slower mean—the shape of the entire distribution changed. This stretching pattern is associated with other cognitive manipulations as well (e.g., Holden, 2002; van Rooij et al., 2013). More generally, the long recognized correlation between the mean and standard deviation of response time distributions (e.g., Luce, 1986; Wagenmakers & Brown, 2007) could arise from this particular pattern of distribution broadening.

Guy always said “listen to your data.” Offering a plausible narrative for the distribution stretching associated with many standard cognitive manipulations is an important challenge both for dynamic and all other perspectives on cognitive performance. A plausible working hypothesis is that rescaling and related distribution shape changes are symptomatic of underlying changes in the dynamics that support cognitive activity. More potential response options implies relatively more competition among the dynamics that support alternate interpretations of presented items. If the dynamics supporting performance are multiplicative, iterative, and perhaps interdependent, then diluting sources of constraint by introducing additional sources of functional ambiguity should sometimes yield patterns of change that approximate proportional changes in dynamics.

A research participant’s job in most word recognition tasks is to supply unique linguistic or semantic acts in response to experimenter supplied letter strings. The fact that word recognition performance is affected by the feed-forward ambiguity of multiple potential pronunciation options for a given spelling-body or whole-word letter string seems a bit unsurprising. In some sense, a response option must be selected, and selection costs time.

By contrast, the fact that a pronunciation rime, or the whole-word pronunciation, of a presented letter string, might be spelled in several ways affects performance is counterintuitive. As Guy said, “In visual lexical decision, the letter string is clearly visible to the participant, and it remains visible until a response is recorded. But if feedback from phonology suggests that some other letter-string could have been presented, recognition is slower” (Van Orden et al., 2001). Among the vast library of cognitive effects in the word recognition liter-
The effect of feedback inconsistency or ambiguity was a particular favorite of Guy’s (Stone et al., 1997). To him, it meant that relations among stimulus forms and linguistic functions must be conceived in terms of a bidirectional and iterative dynamic flow.

As the ratio of balance among cooperative and competitive constraints governing performance approaches unity, the relative impact of intrinsic and extrinsic sources noise will increasingly dominate the circular dynamic flow of perception, cognition, and action (Van Orden, Holden, Podgornik, & Aitchison, 1999). Thus, relative to the absolute clock time of a laboratory computer, cognitive and neurophysiological time express proportional dilation or stretching.

Chemical and biological processes govern all neurophysiological and behavioral activity. These often rate-limited neurochemical and physiological processes do not generally unfold at truly constant rates but rather at variable and often proportional rates across fractal resource networks (e.g., Limpert, Stahel, & Abbt, 2001; Werner, 2010; G. B. West, Brown, & Enquist, 1999). As such, the mismatch between absolute clock time and the relative time of biological activity may explain the phenomena of ubiquitous scaling in biological systems (e.g., D. West & West, 2012). In the end, cognitive activity is a biological function.

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