The lack of the $|2><1|$ makes the operator non-Hermitian. This results in the possibility of the expectation value for $H$ being complex, which is physically meaningless/inadmissible. Now, $\mathcal{U}^\dagger \mathcal{U} \neq 1$. The “conservation of probabilities” is therefore not conserved in this scenario. We should be able to see this in determining the probability of our system being in either of the two given states $|1>$ and $|2>$. Sakurai asks us to ignore all but the final term, $H_{12} = |1><2|$. Using Sakurai 1.15, the time evolution operator now becomes $\mathcal{U} = 1 - \frac{iH_{12}dt}{\hbar}$. We now seek to apply this operator to a general state $|\Psi> = \alpha|1> + \beta|2>$. We find that $H_{12}|\Psi> \rightarrow (1 - \frac{iH_{12}dt}{\hbar})[\alpha|1> + \beta|2>].$ We recover

$$\alpha|1> + \beta|2> - \frac{itH_{12}}{\hbar}\beta|1>.$$ 

Upon inspection, we see that the probability of finding our system in $|1>$ is strange, and violates the rule that $|c_1|^2 = |c_2|^2$, in our case, the $\alpha$ and $\beta$. 

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Sakurai 2.2

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