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Basic global relative invariants for nonlinear differential equations. (English) Zbl 1136.34001

Let be a field of characteristic zero with differentiation \( \theta \), \( y \) be a differential indeterminate and \( f \in F_3 \{ y \} \). For any \( \lambda \in F^* \) put \( u = \lambda^{-1} y \) then \( u \) is also indeterminate over \( F \) and the substitution \( y = \lambda u \) in \( f \) gives a new differential polynomial \( \hat{f} \in F \{ u \} \).

Analogously, for any \( \alpha \in F^* \) put \( \delta = \alpha^{-1} \theta \) then \( \delta \) is also a differentiation of \( F \) and the substitution \( \theta = \alpha \delta \) in \( f \) gives a new differential polynomial \( \tilde{f} \in F \{ y \} \). Under these transformations, with use of a suitable normalizing multiplier, some common algebraic properties of the polynomial \( f \) are kept. For example, if \( f \) is a quadratic form in indeterminates like this

\[
q_m = (\theta^m y)^2 + \sum_{i=1}^{m} c_{ij} \theta^{m-i} y \theta^{m-j} y (i, j = 0, \ldots, m; i + j \neq 0)
\]

with a symmetrical matrix of coefficients \( c_q = (c_{ij}) (c_{ij} \in F) \), then the polynomials \( q_m \) and \( \bar{q}_m \) are also quadratic form in indeterminates \( u, \theta u, \ldots, \theta^m u \) and \( y, \delta y, \ldots, \delta^m y \) (respectively) with the symmetrical matrices of coefficients \( c_q \) and \( \bar{c}_q \), respectively.

So, we get a transformation \( \tau_\lambda (\tau_\alpha) \) on set of matrices over \( F \) mapping \( c_q \mapsto \bar{c}_q \) \( (c_q \mapsto \tilde{c}_q) \) and \( \tau_\lambda (\tau_\alpha) \) is a regular differentially algebraic mapping over \( F \).

Let \( \mathbb{Q} \) be a field of rational numbers and \( \mathbb{Q} \{ w \} \) be a ring of differential polynomials in the differential indeterminates \( w_i (i, j = 0, \ldots, m; i + j \neq 0) \). A differential polynomial \( P \in \mathbb{Q} \{ w \} \) is called relative (or semi-)invariant if \( P(\tau_\lambda (w)) = \alpha^q P(w) \) and \( P(\tau_\alpha (w)) = \alpha^q P(w) \) (for some \( q \in \mathbb{N} \)). For example, the polynomials

\[
L_{211} = w_{11} - (w_{01})^2,
\]
\[
L_{212} = w_{12} - \frac{1}{2} w_{01} w_{11} - w_{01} w_{02} + \frac{1}{4} (w_{01})^3 - \frac{1}{4} w_{11}^{(1)} + \frac{1}{2} w_{01} w_{01}^{(1)}
\]

are two from three of basic relative invariants for \( q_2 \).

In this memoir, as well as in previous (see [Mem. Am. Math. Soc. 744 (2002; Zbl 1006.34084)]), the full description of the basic relative invariants for considered differential polynomials is given. An evaluation of invariants manually is possible only for differential polynomials of the small order and a low degree. Therefore the description of invariants is given in the form of recurrent relations and instructions for machine evaluations in Mathematica system are provided. The exposition is self-contained, well illustrated by examples and based on classical technique.

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MSC:
34-02 Research monographs (ordinary differential equations)
34A34 Nonlinear ODE and systems, general
34M45 Ordinary differential equations on complex manifolds
34M15 Algebraic aspects of ODE in the complex domain

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Software:
Mathematica (http://www.swmath.org/software/554)