INAPPROPRIATE GENERALIZATIONS

When unmotivated abstractions and generalizations are introduced in mathematics, it may be for the purpose of giving someone something to do of a non-demanding nature. History can then be ignored. It can also be a way to avoid explicitly admitting ignorance when consulted for help about concrete problems. Or, perhaps as an occupational deformation, there may be a perceived need to appear brilliant.

Zentralblatt MATH, the online successor to the Zentralblatt fuer Mathematik, has editors that permit their reporters to write obvious nonsense.

As the first of two illustrations, we consider its report Zbl 1006.34084 about my A.M.S. Memoir of 2002 titled Basic Relative Invariants for Homogeneous Linear Differential Equations. A copy of Zbl 1006.34084 can be obtained by clicking here. Without precisely mentioning anything in that memoir, the author of Zbl 1006.34084 mistakenly believed that the main results in that memoir can be immediately generalized to the situation where the coefficients of the differential equations belong to any given ordinary differential field of characteristic zero. In Zbl 1006.34084, he attempted to define with his context a transformation he referred to as (2) that would correspond to a change of the independent variable in classical analysis. After various assertions dependent on (2), he began the last paragraph of his report with the statement: “We note that the substitution (2) is too general. For example, any operator.” Thus, in the last paragraph of Zbl 1006.34084, the author of Zbl 1006.34084 confesses that the generalized context he sought to introduce in Zbl 1006.34084 is inappropriate as he has written it. He concluded by mentioning several other esoteric abstractions that he implies might enable him to be able to write a factually correct report (with coefficients in any ordinary differential field of characteristic zero). I know that those abstractions are also inappropriate for that purpose.

As a second illustration where Zentralblatt MATH communicated nonsense, a copy of the report Zbl 1136.34001 about my 2007 A.M.S. Memoir titled Basic Global Relative Invariants for Nonlinear Differential Equations can be downloaded by clicking here. The author of Zbl 1136.34001 mistakenly believed that my main results can be generalized to the situation where the coefficients of the differential equations belong to any given ordinary differential field $F$ of characteristic zero. With his use of $F^*$ to denote the set of nonzero elements of any such field $F$, he began his report by implying that the transformations corresponding to changes of the independent variable in a classical context are satisfactorily specified in his context by the nonzero elements of $F$. That is clearly incorrect. Namely, consider the situation where $F$ is the ordinary differential field of meromorphic functions on a region $U$ of the complex plane and the equation $H = 0$ to be transformed has meromorphic coefficients on $U$. Then, the global transformations of $H = 0$ that correspond to changes of the independent variable are precisely known to be uniquely specified by the univalent analytic functions on $U$. In other words, they are only specified by the nonzero meromorphic functions on $U$ that also happen to be univalent analytic functions on $U$. His inapplicable context makes most of Zbl 1136.34001 unintelligible.

My third monograph on relative invariants, titled Relative Invariants from 1879 Onward: Their Evolution for Differential Equations, builds on the results of my two memoirs and is primarily concerned with the development of new identities with which to combine basic relative invariants in order to explicitly construct relative invariants having a given weight. However, Section 2.5 on page 24 of that monograph, titled Avoidance of inappropriate abstractions, examines in more detail the inadvisability of seeking context generalizations particularly when there is much yet to be learned of a nontrivial nature.