Linear Algebra MATH 2076 Quiz-5

Be sure to show your work. **No credit for inspired answers!**

1. Assume that

\[
A = \begin{bmatrix}
1 & -1 & 1 & -1 & 1 \\
2 & -2 & 2 & -2 & 3 \\
1 & -1 & 2 & -2 & 3 \\
1 & -1 & 1 & -1 & 2
\end{bmatrix}
\]

is row equivalent\(^1\) to

\[
B = \begin{bmatrix}
1 & -1 & 0 & 0 & 1 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Find bases for the null space \(\text{Nul}(A)\) and the column space \(\text{Col}(A)\), and give their dimensions.

**Answer:** Compare Exercises 23-26 in Section 2.8

We note that \(B\) in the echelon form. For a basis of \(\text{Col}(A)\) we take the pivot columns, that is columns 1, 3, 5.

This gives a basis \(\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 3 \\ 2 \\ 1 \end{bmatrix} \right\}\). The dimension of \(\text{Col}(A)\) is 3.

To find the null space, we solve \(A\vec{x} = 0\) which is the same as \(B\vec{x} = 0\). The free variables are \(x_2, x_4\) and the general solution is \(x_1 = x_2, x_2 = x_2, x_3 = x_4, x_4 = x_4, x_5 = 0\).

So \(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}\).

The basis of \(\text{Nul}(A)\) is \(\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}\). The dimension of \(\text{Nul}(A)\) is 2.

\(^1\)Do not verify that \(B\) was computed correctly - let’s trust my computer here.