1. Determine the general solution of the equation $y^{iv} - y = 12e^t$

The characteristic equation $r^4 - 1 = 0$ factors as $r^4 - 1 = (r^2 - 1)(r^2 + 1) = (r - 1)(r + 1)(r - i)(r + i) = 0$ with roots $\pm 1, \pm i$. The general solution of homogeneous equation is $y = C_1e^t + C_2e^{-t} + C_3\sin t + C_4\cos t$.

We seek particular solution $y = tAe^t$ and get $A = 3$. Answer: $\{y = C_1e^t + C_2e^{-t} + C_3\sin t + C_4\cos t + 3te^t\}$

2. Solve the initial value problem $y^{iii} + 4y' = 6\cos t$ with $y(0) = 0$, $y'(0) = 2$, $y''(0) = 0$.

**Solution:** Characteristic equation $r^3 + 4r = 0$ factors as $r(r - 2i)(r + 2i) = 0$. The roots are $0, -2i, 2i$. The general solution of the homogeneous equation is $y = C_1 + C_2\cos 2t + C_3\sin 2t$.

We seek particular solution $y_p = A\cos t + B\sin t$ and get $A = 0, B = 2$. So we have general solution $y = C_1 + C_2\cos 2t + C_3\sin 2t + 2\sin t$.

The initial conditions give

$$C_1 + C_2 = 0$$
$$2C_3 + 2 = 2$$
$$-4C_2 = 0$$

The answer is $\{y(t) = 2\sin t\}$

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We seek particular solution $y = tAe^t$ and get $A = 3$. Answer: $\{y = C_1e^t + C_2e^{-t} + C_3\sin t + C_4\cos t + 3te^t\}$

2. Solve the initial value problem $y^{iii} + 9y' = 8\cos t$ with $y(0) = -1$, $y'(0) = 0$, $y''(0) = 1$.

**Solution:** Characteristic equation $r^3 + 9r = 0$ factors as $r^2(r - 3i)(r + 3i) = 0$. The roots are $0, -3i, 3i$. The general solution of the homogeneous equation is $y = C_1 + C_2\cos 3t + C_3\sin 3t$.

Using undetermined coefficients, $y = A\cos t + B\sin t$ we get $A = 0, B = 1$. So the general solution of non-homogeneous equation is $y = C_1 + C_2\cos 3t + C_3\sin 3t + \sin t$.

The initial conditions give

$$C_1 + C_2 = -1$$
$$3C_3 + 1 = 0$$
$$-9C_2 = 1$$

The answer is $\{y(t) = -\frac{8}{9} - \frac{1}{3}\cos(3t) - \frac{1}{3}\sin 3t + \sin t\}$