Differential Equations MATH 2073 Quiz-5 \_key

**Instructions.** Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Solve the initial value problem $y'' - 4y = 6e^t$, $y(0) = 7$, $y'(0) = 0$.
   
   The homogeneous equation $y'' - 4y = 0$ has characteristic equation $r^2 - 4r = 0$ with roots $r = \pm 2$ that give $y_1 = e^{2t}, y_2 = e^{-2t}$.
   
   Using method of undetermined parameters, we seek a particular solution $y_\ast = Ae^t$. We compute $A = -2$.
   
   So the formula $y = C_1y_1 + C_2y_2 + y_\ast$ for the general solution gives $y(t) = C_1e^{2t} + C_2e^{-2t} - 2e^t$.
   
   Using the initial condition we now compute
   
   $$y(0) = C_1 + C_2 - 2 = 7$$
   $$y'(0) = 2C_1 - 2C_2 - 2 = 0$$
   
   We get $C_1 = 5, C_2 = 4$.
   
   The answer is $y(t) = 5e^{2t} + 4e^{-2t} - 2e^t$.

2. Find the general solution of nonhomogeneous equation $y'' - 4y' + 4y = 6t$.
   
   We first solve homogeneous equation $y'' - 4y' + 4y = 0$. The characteristic equation is $r^2 - 4r + 4 = (r - 2)^2 = 0$, with double root. The general solution of homogeneous equation is $y = C_1e^{2t} + C_2te^{2t}$.
   
   By undetermined coefficients method we seek particular solution of the form $y_\ast = A + Bt$. We get $B = \frac{3}{2}, A = B = \frac{3}{2}$ so $y_\ast = \frac{3}{2}(1 + t)$.
   
   Combining these two together, we get the answer: $y = C_1e^{2t} + C_2te^{2t} + \frac{3}{2}(1 + t)$.
Differential Equations MATH 2073 Quiz-5 \textbf{Key}

\textbf{Instructions.} Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Solve the initial value problem \( y'' - 4y = 18e^t \), \( y(0) = 5 \), \( y'(0) = 0 \).

   The homogeneous equation \( y'' - 4y = 0 \) has characteristic equation \( r^2 - 4r = 0 \) with roots \( r = \pm 2 \) that give \( y_1 = e^{2t} \), \( y_2 = e^{-2t} \).

   Using method of undetermined parameters, we seek a particular solution \( y_* = Ae^t \). We compute \( A = -6 \).

   So the formula \( n C_1 y_1 + C_2 y_2 + y_* \) for the general solution gives \( y(t) = C_1 e^{2t} + C_2 e^{-2t} - 6e^t \).

   Using the initial condition we now compute

   \[
   \begin{align*}
   y(0) &= C_1 + C_2 - 6 = 5 \\
   y'(0) &= 2C_1 - 2C_2 - 6 = 0
   \end{align*}
   \]

   We get \( C_1 = 7, C_2 = 4 \).

   The answer is \( y(t) = 7e^{2t} + 4e^{-2t} - 6e^t \).

2. Find the general solution of nonhomogeneous equation \( y'' - 4y' + 4y = 10t \).

   We first solve homogeneous equation \( y'' - 4y' + 4y = 0 \). The characteristic equation is \( r^2 - 4r + 4 = (r - 2)^2 = 0 \), with double root. The general solution of homogeneous equation is \( y = C_1 e^{2t} + C_2 te^{2t} \).

   By undetermined coefficients method we seek particular solution of the form \( y_* = A + Bt \). We get \( B = \frac{5}{2}, A = B = \frac{5}{2} \) so \( y_* = \frac{5}{2}(1 + t) \).

   Combining these two together, we get the answer: \( y = C_1 e^{2t} + C_2 te^{2t} + \frac{5}{2}(1 + t) \).
Differential Equations MATH 2073 Quiz-5c \textbf{key}

\textbf{Instructions.} Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Solve the initial value problem \( y'' - 4y = 12e^t, \ y(0) = 2, \ y'(0) = 0. \)

The homogeneous equation \( y'' - 4y = 0 \) has characteristic equation \( r^2 - 4r = 0 \) with roots \( r = \pm 2 \) that give \( y_1 = e^{2t}, \ y_2 = e^{-2t}. \)

Using method of undetermined parameters, we seek a particular solution \( y_\ast = Ae^t. \) We compute \( A = -4. \)

So the formula \( y = C_1y_1 + C_2y_2 + y_\ast \) for the general solution gives \( y(t) = C_1e^{2t} + C_2e^{-2t} - 4e^t. \)

Using the initial condition we now compute

\[
\begin{align*}
y(0) &= C_1 + C_2 - 4 = 2 \\
y'(0) &= 2C_1 - 2C_2 - 4 = 0
\end{align*}
\]

We get \( C_1 = 4, C_2 = 2. \)

The answer is \( \{y(t) = 4e^{2t} + 2e^{-2t} - 4e^t\}. \)

2. Find the general solution of nonhomogeneous equation \( y'' - 4y' + 4y = 8t. \)

We first solve homogeneous equation \( y'' - 4y' + 4y = 0. \) The characteristic equation is \( r^2 - 4r + 4 = (r - 2)^2 = 0, \) with double root. The general solution of homogeneous equation is \( y = C_1e^{2t} + C_2te^{2t}. \)

By undetermined coefficients method we seek particular solution of the form \( y_\ast = A + Bt. \) We get \( B = 2, \ A = B = 2 \) so \( y_\ast = 2(1 + t). \)

Combining these two together, we get the answer: \( \{y = C_1e^{2t} + C_2te^{2t} + 2(1 + t)\}. \)
Differential Equations MATH 2073 Quiz-5 Key

Instructions. Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Solve the initial value problem \( y'' - 4y = 12e^t, \ y(0) = 4, \ y'(0) = 0. \)

   The homogeneous equation \( y'' - 4y = 0 \) has characteristic equation \( r^2 - 4r = 0 \) with roots \( r = \pm 2 \) that give \( y_1 = e^{2t}, \ y_2 = e^{-2t}. \)

   Using method of undetermined parameters, we seek a particular solution \( y_* = Ae^t. \) We compute \( A = -4. \)

   So the formula \( n \ C_1 y_1 + C_2 y_2 + y_* \) for the general solution gives \( y(t) = C_1 e^{2t} + C_2 e^{-2t} - 4e^t. \)

   Using the initial condition we now compute

   \[
   y(0) = C_1 + C_2 - 4 = 4 \\
   y'(0) = 2C_1 - 2C_2 - 4 = 0
   \]

   We get \( C_1 = 5, \ C_2 = 3. \)

   The answer is \( y(t) = 5e^{2t} + 3e^{-2t} - 4e^t. \)

2. Find the general solution of nonhomogeneous equation \( y'' - 4y' + 4y = 5t. \)

   We first solve homogeneous equation \( y'' - 4y' + 4y = 0. \) The characteristic equation is \( r^2 - 4r + 4 = (r - 2)^2 = 0, \) with double root. The general solution of homogeneous equation is \( y = C_1 e^{2t} + C_2 te^{2t}. \)

   By undetermined coefficients method we seek particular solution of the form \( y_* = A + Bt. \) We get \( B = \frac{5}{4}, \ A = B = \frac{5}{4} \) so \( y_* = \frac{5}{4}(1 + t). \)

   Combining these two together, we get the answer: \( y = C_1 e^{2t} + C_2 te^{2t} + \frac{5}{4}(1 + t). \)