Differential Equations MATH 2073 Quiz-3 \( \text{key} \)

**Instructions.** Be sure to show your work so that it is clear how you got your answers.

1. Determine the equilibrium points of \( y' = (y - 1)(5 - y) \), and classify each as asymptotically stable or unstable. 
   \( (y - 1)(5 - y) = 0 \) when \( y = 1 \) (unstable) or \( y = 5 \) (stable).

   \[
   (y - 1)(5 - y) = 0 \quad \text{when} \quad y = 1 \quad \text{(unstable)} \quad \text{or} \quad y = 5 \quad \text{(stable)}.
   \]

2. Find the general solution of \( y' = (y - 1)(5 - y) \). Write your answer in explicit form.

   This is a separable equation.
   \[
   \frac{dy}{(y - 1)(y - 5)} = -dt
   \]

   Partial fractions:
   \[
   \left( \frac{1}{y - 5} - \frac{1}{y - 1} \right) dy = -4dt
   \]

   Integrating this, we get:
   \[
   \ln |y - 5| - \ln |y - 1| = -4t + c
   \]

   Now some algebra to get the solution
   \[
   \frac{y - 5}{y - 1} = Ce^{-4t}
   \]
   \[
   y - 5 = Ce^{-4t}(y - 1)
   \]
   \[
   y(1 - Ce^{-4t}) = 5 - Ce^{-4t}
   \]

   Answer: \( y = \frac{5 - Ce^{-4t}}{1 - Ce^{-4t}} \)
Differential Equations MATH 2073 Quiz-3 Key

Instructions. Be sure to show your work so that it is clear how you got your answers.

1. Determine the equilibrium points of \( y' = (y - 1)(4 - y) \), and classify each as asymptotically stable or unstable.
   \((y - 1)(4 - y) = 0\) when \(y = 1\) (unstable) or \(y = 4\) (stable).

2. Find the general solution of \( y' = (y - 1)(4 - y) \). Write your answer in explicit form.
   This is a separable equation.
   \[
   \frac{dy}{(y - 1)(y - 4)} = -dt
   \]
   Partial fractions:
   \[
   \left(\frac{1}{y - 4} - \frac{1}{y - 1}\right) dy = -3dt
   \]
   Integrating this, we get:
   \[
   \ln |y - 4| - \ln |y - 1| = -3t + c
   \]
   Now some algebra to get the solution:
   \[
   \frac{y - 4}{y - 1} = Ce^{-3t}
   \]
   \[
   y - 4 = Ce^{-3t}(y - 1)
   \]
   \[
   y(1 - Ce^{-3t}) = 4 - Ce^{-3t}
   \]
   Answer: \( y = \frac{4 - Ce^{-3t}}{1 - Ce^{-3t}} \)
Differential Equations MATH 2073 Quiz-3 \text{ Key}

Instructions. Be sure to show your work so that it is clear how you got your answers.

1. Determine the equilibrium points of \( y' = (y - 1)(3 - y) \), and classify each as asymptotically stable or unstable.
   \((y - 1)(3 - y) = 0\) when \( y = 1 \) (unstable) or \( y = 3 \) (stable).

2. Find the general solution of \( y' = (y - 1)(3 - y) \). Write your answer in explicit form.
   This is a separable equation.
   \[
   \frac{dy}{(y - 1)(y - 3)} = -dt
   \]
   Partial fractions:
   \[
   \left( \frac{1}{y - 3} - \frac{1}{y - 1} \right) dy = -2dt
   \]
   Integrating this, we get:
   \[
   \ln |y - 3| - \ln |y - 1| = -2t + c
   \]
   Now some algebra to get the solution
   \[
   \frac{y - 3}{y - 1} = Ce^{-2t}
   \]
   \[
   y - 3 = Ce^{-2t}(y - 1)
   \]
   \[
   y(1 - Ce^{-2t}) = 3 - Ce^{-2t}
   \]
   Answer: \[
   y = \frac{3 - Ce^{-2t}}{1 - Ce^{-2t}}
   \]
Differential Equations MATH 2073 Quiz-3 \_key

Instructions. Be sure to show your work so that it is clear how you got your answers.

1. Determine the equilibrium points of \( y' = (y - 1)(6 - y) \), and classify each as asymptotically stable or unstable.
   
   \( (y - 1)(6 - y) = 0 \) when \( y = 1 \) (unstable) or \( y = 6 \) (stable).

2. Find the general solution of \( y' = (y - 1)(6 - y) \). Write your answer in explicit form.
   This is a separable equation.

\[
\frac{dy}{(y - 1)(y - 6)} = -dt
\]

Partial fractions:

\[
\left( \frac{1}{y - 6} - \frac{1}{y - 1} \right) dy = -5dt
\]

Integrating this, we get:

\[
\ln |y - 6| - \ln |y - 1| = -5t + c
\]

Now some algebra to get the solution

\[
y - 6 = Ce^{-5t}
\]

\[
y(1 - Ce^{-5t}) = 6 - Ce^{-5t}
\]

Answer: \( y = \frac{6 - Ce^{-5t}}{1 - Ce^{-5t}} \)