Differential Equations MATH 2073 Exam-1 Key

Instructions. In the differential equations below, \( y = y(t) \) is a function of independent variable \( t \). Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Identify all applicable methods that could be used to solve the equation for \( y \) as the function of \( t \). Put a check \( \sqrt{\ } \) in every box that is applicable.

<table>
<thead>
<tr>
<th>No</th>
<th>Equation</th>
<th>Linear?</th>
<th>Separable?</th>
<th>None of these?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( y' = y + 1 )</td>
<td>( \sqrt{\ } )</td>
<td>( \sqrt{\ } )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( y' = \frac{1}{t+1} )</td>
<td>( \sqrt{\ } )</td>
<td>( \sqrt{\ } )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( y' = ty )</td>
<td>( \sqrt{\ } )</td>
<td>( \sqrt{\ } )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( y' = e^x + y )</td>
<td>( \sqrt{\ } )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>( y' = ty(1 - y) )</td>
<td>( \sqrt{\ } )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>( y' = y(t - y) )</td>
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<td>( \sqrt{\ } )</td>
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</tr>
<tr>
<td>7</td>
<td>( y' + ty = e^y )</td>
<td></td>
<td>( \sqrt{\ } )</td>
<td></td>
</tr>
</tbody>
</table>

Note: For equations that can be solved by both methods be sure to mark both. Some equations cannot be solved by these two methods!!

2. Solve the initial value problem \( y' = y + 1; \ y(0) = 0 \) (This is the first equation in Problem 1.)

Solution This is a linear and a separable equation, so both methods can be used. The general solution is \( y = -1 + Ce^x \) so the answer is \( y(x) = e^x - 1 \).

3. Solve the initial value problem \( y' = \frac{1}{x+1}; \ y(0) = 2 \) for the unknown function \( y = y(x) \) of \( x \geq 0 \).

(This is the second equation in Problems 1!) Solution This is a separable and a linear equation. Integrating both sides, the general solution is \( y = C + \ln(1 + x) \) so the answer is \( y(x) = 2 + \ln(1 + x) \).
4. Find the general solution of \( y' = ty \)

(This is the third equation in Problem 1.) **Solution** This equation can be solved as linear, or as separable.

\[
\frac{dy}{y} = x \, dx
\]

\[
\ln|y| = \frac{x^2}{2} + c
\]

**Answer:** \( y = Ce^{x^2/2} \)

5. Find the general solution of the equation \( y' = e^t + y \)

(This is the fourth equation in Problem 1.) **Solution** This is linear equation \( y' - y = e^x \) with integrating factor \( e^{-x} \).

\[
e^{-x}y' - ye^{-x} = 1
\]

\[
(e^{-x}y)' = 1
\]

\[
e^{-x}y = x + C
\]

So the answer is \( y = xe^x + Ce^x \) where \( C \) is an arbitrary constant.

6. Match the directional fields with the differential equations A, B, C below. (use arrows to indicate your matches)

<table>
<thead>
<tr>
<th>Directional Field</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Directional Field 1" /></td>
<td>( y' = ty(1 - y) ) B</td>
</tr>
<tr>
<td><img src="image2.png" alt="Directional Field 2" /></td>
<td>( y' = y(t - y) ) C</td>
</tr>
<tr>
<td><img src="image3.png" alt="Directional Field 3" /></td>
<td>( y' = y(1 - y) ) A</td>
</tr>
</tbody>
</table>

**A:** \( y' = y(1 - y) \)

**B:** \( y' = ty(1 - y) \)

**C:** \( y' = y(t - y) \)
Differential Equations MATH 2073 Exam-1

Instructions. In the differential equations below, $y = y(t)$ is a function of independent variable $t$. Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Identify all applicable methods that could be used to solve the equation for $y$ as the function of $t$. Put a check $\sqrt{}$ in every box that is applicable.

<table>
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<tr>
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<td>$\sqrt{}$</td>
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<tr>
<td>2</td>
<td>$y' = \frac{1}{t+1}$</td>
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<tr>
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<td>$y' = 4ty$</td>
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Note: For equations that can be solved by both methods be sure to mark both. Some equations cannot be solved by these two methods!!!

2. Solve the initial value problem $y' = y - 1; y(0) = 2$

Solution This is a linear and a separable equation, so both methods can be used. The general solution is $y = 1 + Ce^x$ so the answer is $y(x) = 1 + e^x$.

3. Solve the initial value problem $y' = \frac{1}{x+t}; y(0) = 3$ for the unknown function $y = y(x)$ of $x \geq 0$.

Solution This is a separable and a linear equation. Integrating both sides, the general solution is $y = C + \ln(1 + x)$ so the answer is $y(x) = 3 + \ln(1 + x)$. 
4. Find the general solution of \( y' = 4ty \)

**Solution** This equation can be solved as linear, or as separable.

\[
\frac{dy}{y} = 4tdx
\]

\[
\ln|y| = 2x^2 + c
\]

Answer: \( y = Ce^{2x^2} \)

5. Find the general solution of the equation \( y' = e^t + y \)

**Solution** This is linear equation \( y' - y = e^x \) with integrating factor \( e^{-x} \).

\[
e^{-x}y' - ye^{-x} = 1
\]

\[
(e^{-x}y)' = 1
\]

\[
e^{-x}y = x + C
\]

So the answer is \( y = xe^x + Ce^x \) where \( C \) is an arbitrary constant.

6. Match the directional fields with the differential equations A, B, C below. (use arrows to indicate your matches)

- **This is** \( y' = y(t - y) \) (C)
- **This is** \( y' = ty(1 - y) \) (B)
- **This is** \( y' = y(1 - y) \) (A)
Differential Equations MATH 2073 Exam-1 Key

Instructions. In the differential equations below, $y = y(t)$ is a function of independent variable $t$. Simplify your answers when appropriate. Be sure to show your work so that it is clear how you got your answers.

1. Identify all applicable methods that could be used to solve the equation for $y$ as the function of $t$. Put a check $\checkmark$ in every box that is applicable.

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2. Solve the initial value problem $y' = y + 1; y(0) = 0$  Solution This is a linear and a separable equation, so both methods can be used. The general solution is $y = -1 + Ce^x$ so the answer is $y(x) = e^x - 1$.

3. Solve the initial value problem $y' = \frac{1}{x+1}; y(0) = 4$ for the unknown function $y = y(x)$ of $x \geq 0$.

Solution This is a separable and a linear equation. Integrating both sides, the general solution is $y = C + \ln(1 + x)$ so the answer is $y(x) = 4 + \ln(1 + x)$. 


4. Find the general solution of $y' = ty$

**Solution** This equation can be solved as linear, or as separable.

$$\frac{dy}{y} = xdx$$

$$\ln |y| = \frac{x^2}{2} + c$$

Answer: $y = Ce^{x^2/2}$

5. Find the general solution of the equation $y' = e^x + y$

**Solution** This is linear equation $y' - y = e^x$ with integrating factor $e^{-x}$.

$$e^{-x}y' - ye^{-x} = 1$$

$$(e^{-x}y)' = 1$$

$$e^{-x}y = x + C$$

So the answer is $y = xe^x + Ce^x$ where $C$ is an arbitrary constant.

6. Match the directional fields with the differential equations A, B, C below. (use arrows to indicate your matches)

- This is $y' = y(1 - y)$ A
- This is $y' = ty(1 - y)$ B
- This is $y' = y(t - y)$ C
Differential Equations MATH 2073 Exam-1 \textit{Key}

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\textbf{Solution} This is a linear and a separable equation, so both methods can be used. The general solution is \( y = 1 + Ce^x \) so the answer is \( y(x) = 1 + e^x \).

3. Solve the initial value problem \( y' = \frac{1}{x+1}; \ y(0) = 5 \) for the unknown function \( y = y(x) \) of \( x \geq 0 \).

\textbf{Solution} This is a separable and a linear equation. Integrating both sides, the general solution is \( y = C + \ln(1 + x) \) so the answer is \( y(x) = 5 + \ln(1 + x) \).
4. Find the general solution of \( y' = 4ty \)

**Solution** This equation can be solved as linear, or as separable.

\[
\frac{dy}{y} = 4tx\,dx
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\ln |y| = 2x^2 + c
\]

Answer: \( y = Ce^{2x^2} \)

5. Find the general solution of the equation \( y' = e^t + y \)

(This is the fourth equation in Problem 1.) **Solution** This is linear equation \( y' - y = e^t \) with integrating factor \( e^{-x} \).

\[
e^{-x}y' - ye^{-x} = 1
\]

\[
(e^{-x}y)' = 1
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e^{-x}y = x + C
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So the answer is \( y = xe^x + Ce^x \) where \( C \) is an arbitrary constant.

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- **B**: \( y' = ty(1-y) \)
- **C**: \( y' = y(t-y) \)