Instructions. Answer the following.

1. Solve the following differential equations.

   (a) $y' = y + 1; \ y(0) = 0$

   (b) $y' = \frac{x^2 - 1}{y^2 + 1}; \ y(0) = 0$

   (c) $(x + y) - (x - y)y' = 0$

2. Find the solution of the equation $y'' - 2y' = 4$ with the initial value $y(0) = 1, y'(0) = 1$.

3. (a) Find the recurrence relation for the power series solution $y = \sum_{n=0}^{\infty} a_n x^n$ of equation $y'' - xy' + 4y = 0$ with the initial value $y(0) = 1, y'(0) = 1$. 


(b) Find the first six terms in the expansion \( y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots \) for the solution \( y \) of the differential equation \( y'' - xy' + 4y = 0 \) with the initial value \( y(0) = 1, y'(0) = 1 \).

4. Suppose \( x > 0 \).

(a) Find the general solution of the homogeneous Euler equation \( x^2 y'' - xy' + y = 0 \).

(b) Find the general solution of \( x^2 y'' - xy' + y = 2x \).

5. Sketch the graph of the function \( f(t) \) with Laplace transform

\[
\begin{align*}
(a) \quad F(s) &= \frac{2}{s(s^2+1)} \\
(b) \quad F(s) &= \frac{1-e^{-2s}}{s^3}
\end{align*}
\]

6. Solve the differential equation \( y'' = 1 - u_{c}(t) \) with initial condition \( y(0) = y'(0) = 0 \), where \( u_{c}(t) = \begin{cases} 
0 & \text{if } t \leq c \\
1 & \text{if } t > c
\end{cases} \)