Cosmic Microwave Background (CMB) Polarization: How Foreground Dust Correlates across Frequency

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1 Abstract

For this project, our goal is to examine Cosmic Microwave Background radiation (or CMB) polarization. To this end, we will be examining 3 different foreground dust maps to see if these polarized map patterns change with frequency.

To accomplish this, we will be generating foreground maps in Pysm 3 at varying frequencies and then applying a mask to them. From here we may determine the map's E modes and B modes, which will allow us to figure out the map's power spectra, auto spectra, and cross spectra. We may use the maps spectra to examine their correlation. Thus revealing if Dust decorrelates across Frequency.

2 Theory

2.1 Polarization

When we examine the sky we can observe several varying temperature patterns when considering a location outside of the galactic plane. These temperature patterns are directly related to the polarization at that specific point. For this, we will be using Stokes parameters to describe the polarization of electromagnetic radiation, due to the complex galactic magnetic field. The two parameters we will be using are Q polarization and U polarization. Q polarization is associated with 'E-modes' or the scalar fluctuations that affect the CMB temperature. These 'E-modes' are patterns that make the polarization direction parallel to the wave direction(Planck pg.13). The weak gravitational lensing of the 'E-modes produce a 'B-mode' pattern. These rotate the polarization direction by $+45^o$ or -45^o with respect to the wave direction (Planck pg.13). Specifically, Q and U create varying polarization across all of the points in the universe. The polarization variations directly affect the light that travels through the point, therein affecting the resulting temperature. For a more in-depth look at the CMB polarization measurements and analysis, I recommend consulting Samtleben, Staggs, and Winstein's paper titled "The Cosmic Microwave Background for Pedestrians: A Review for Particle and Nuclear Physicists". The paper not only goes into detail about the CMB itself but also brings in important information regarding polarization and temperature fields.

2.2 Foregrounds

The ultimate goal of this project is to be able to examine CMB polarization and temperature across the sky, so why are we examining foreground dust? Whenever we look at temperature distributions across the sky we are seeing a multitude of temperature sources that end up affecting the image we are attempting to view. We call these variables that affect the image foregrounds. There are four primary foregrounds that can affect the resulting temperature, these are Dust (d), Synchrotron (s), Anomalous microwave emission (a), and free-free electrons (f). In a paper from Planck, titled "Planck 2015 results: X. Diffuse component separation: Foreground maps", we see that of these foregrounds Anomalous microwave emission and free-free electrons are theorized unpolarized signals. As such we do not need to consider them, as we are discussing polarization.

2.2.1 Foreground Dust and Synchrotron

This leaves us with dust and synchrotron foregrounds which are covering up the polarization of the CMB. Foreground dust can be described as sand or other space 'dust' which is made up of different elements of solid particles. This dust, when exposed to the galactic magnetic field, gets polarized causing its speed to be affected. Foreground Synchrotron is radiation and free electrons. From Planck, we can see that these foregrounds vary drastically with frequency. When we look at dust, its temperature intensity increases as the frequency is increased. Whereas with synchrotron, as frequency is increased the temperature intensity decreases. If we were to plot these relations we would see a small frequency region where both dust and synchrotron are below the CMB amplitude. Meaning that for some select frequencies, the majority of the temperature we would be seeing would be from the CMB.

If we are able to establish a relationship between the 'dust and frequency', and 'synchrotron and frequency'. Then we may establish a "template" that will allow us to remove dust and sychrotron from the foreground. Leaving us with the polarization of light from the CMB. As such our goal is to see if such a template is possible, which would allow further analysis of the CMB.

2.3 Template Cleaning

Template cleaning is a process in which we can see the correlation (ϵ) between parameters by comparing two maps at varying frequencies. Let us establish a value 'm_n' to be a map (m) value at some n frequency, additionally let γ be the frequency close to dust. Let us set a sample map at $m_{100} = (cmb + dust + sync)$, where sync is synchrotron. To view its correlation or auto spectra, let us take $m_{100} \times m_{100} = (cmb^2 + dust^2 + sync^2 + \epsilon(dust \times sync))$. Here by Planck, our ϵ may be defined as $\epsilon = \Delta log^2(\frac{v_1}{v_2})$, where v_1 and v_2 are the frequencies of two different maps.

Perfect correlation is a case where the two frequency maps correlate perfectly. This can be defined by the following function:

$$\frac{m_1 \times m_2}{\sqrt{(m_1 \times m_1)(m_2 \times m_2)}} = 1$$

When we examine our data, the noticeable downward shift of the auto spectra and cross spectra are referred to as Decorrelation. Which will be explored more below.

2.4 Angular Power Spectra

For angular power spectra to start, we will want to take a Fourier transform of the map, which is just essentially a spherical harmonic transform. Of which we will receive some wavelength (λ), the angular scale (l), and the direction(m). In this case, we do not care about the direct, our main factors are the wavelength and angular scale. What we will see is that as the wavelength is longer the angular scale will be smaller, and vice versa.

For its practical application, we would be applying the operation, map2alm to a masked foreground map. Which will be explored more later on.

3 Activities and Findings

3.1 Generating Foreground Maps

We obtain our dust maps from pysm 3. This is done based on Planck's projected models in "Planck 2015 results: X. Diffuse component separation: Foreground maps". Pysm 3 is a program that generates full-sky simulations of galactic emissions in intensity and polarization. From pysm, our most basic dust map that could be examined is d1. Which is thermal dust modeled as a single component modified black body (mbb). However, for the scope of this project, we will be examining 3 different dust maps: d9, d10, and d12.

d10 is a single component modified black body model based on template projections established by Planck data. This reduces contamination from CIB and point sources when compared to d1.

d9 is a simplified version of the d10 model, which contains a fixed spectral index of 1.48 and a fixed dust black body temperature of 19.6 K all over the sky. It is also based on Planck 2018 results.

d12 is a 3D model of polarized dust emission with 6 layers that is based on a paper by Jacques Delabrouille, titled "A 3-D model of polarized dust emission in the Milky Way".

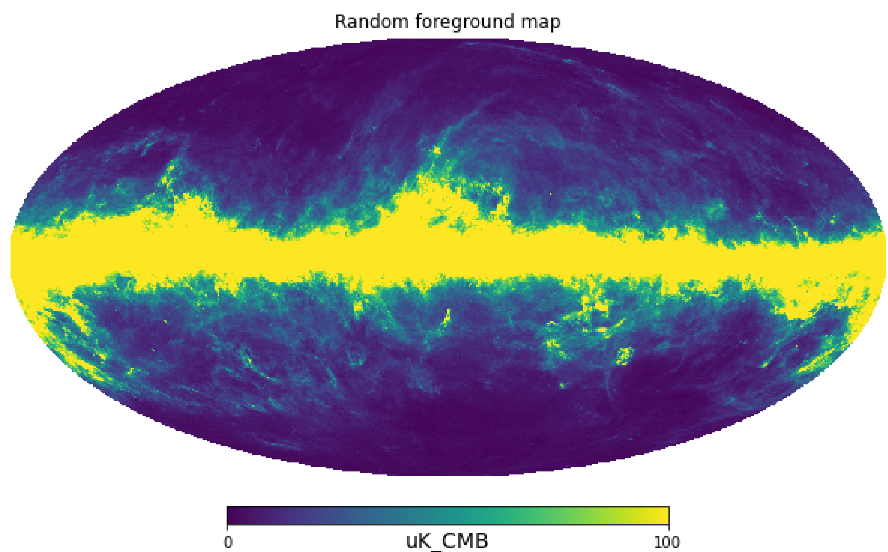

Figure 1: Includes a randomly generated foreground, that gives the reader a general idea of the maps we will be using.

This will allow us to examine the correlation across more complex foreground projections, which will give us an idea of the correlation for more complex foregrounds.

Our goal is to see how dust will correlate across different frequencies. As such once we obtain the foregrounds we will apply varying frequencies to each foreground map. In this case, we will evaluate the maps at the following frequencies: 30 GHz, 40 GHz, 85 GHz, 95 GHz, 145 GHz, 155 GHz, and 220 GHz

Each of these individual frequency-applied foregrounds will generate three maps. These maps are: Temperature in terms of micro-kelvin Rayleigh jeans, Polarization Q, and Polarization U.

3.2 Generating a mask

Each of these maps is in galactic coordinates. For our first step, we want to eliminate the saturated portion of the map or the part that is along the galactic plane. This will allow us to look at regions of the map where the fluctuations due to dust won't be overshadowed by the light of our galaxy. In order to do this we will need to rotate the map out of galactic coordinates and into celestial coordinates.

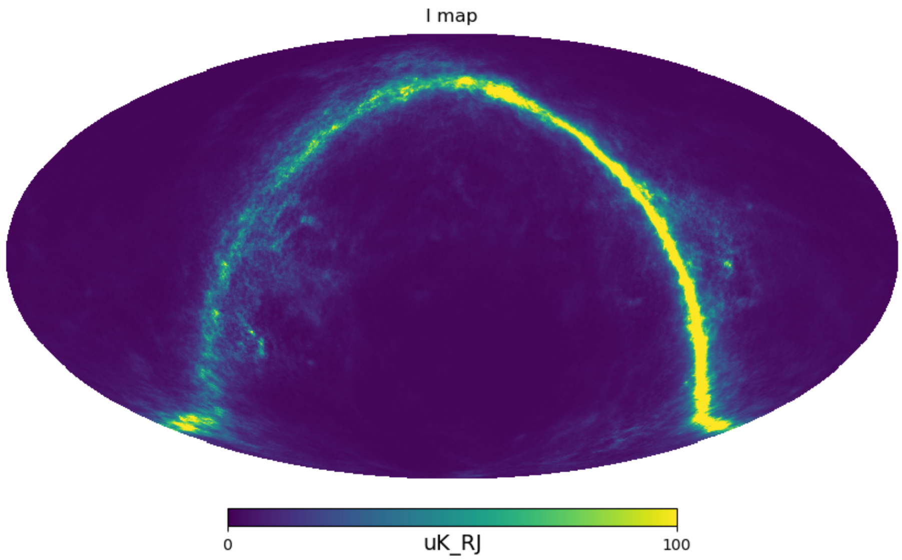

Figure 2: From the d9 foreground, this graph shows the foreground in celestial coordinates at a 30 GHz frequency.

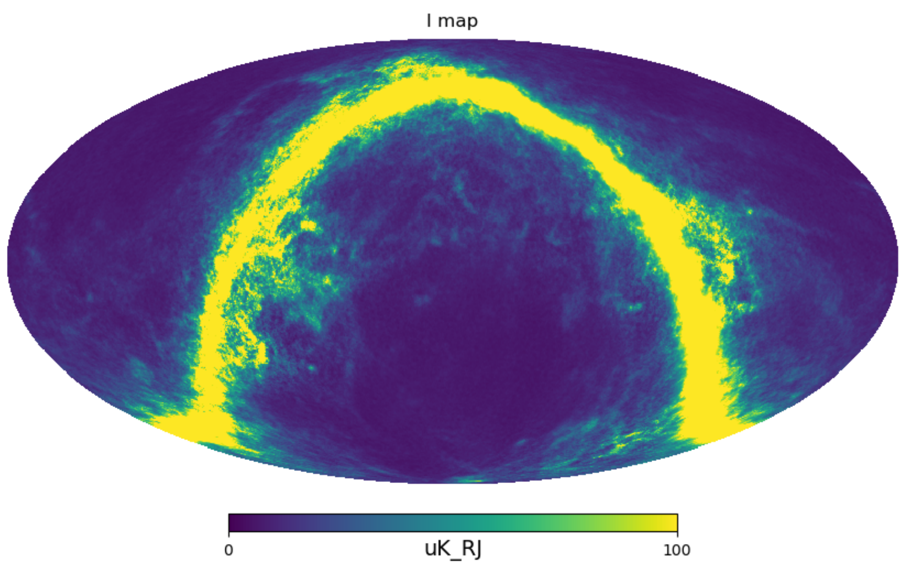

Figure 3: From the d9 foreground, this graph shows the foreground in celestial coordinates at a 85 GHz frequency.

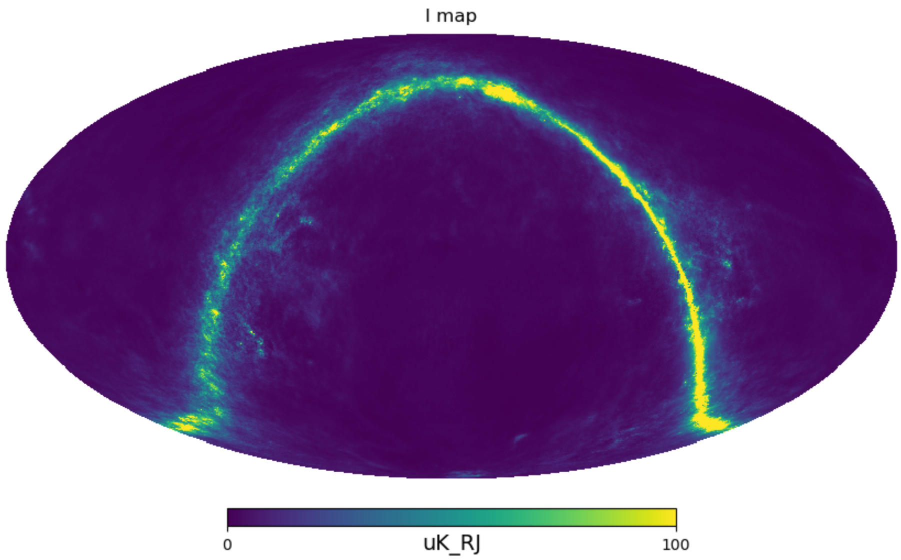

Figure 4: From the d10 foreground, this graph shows the foreground in celestial coordinates at a 30 GHz frequency.

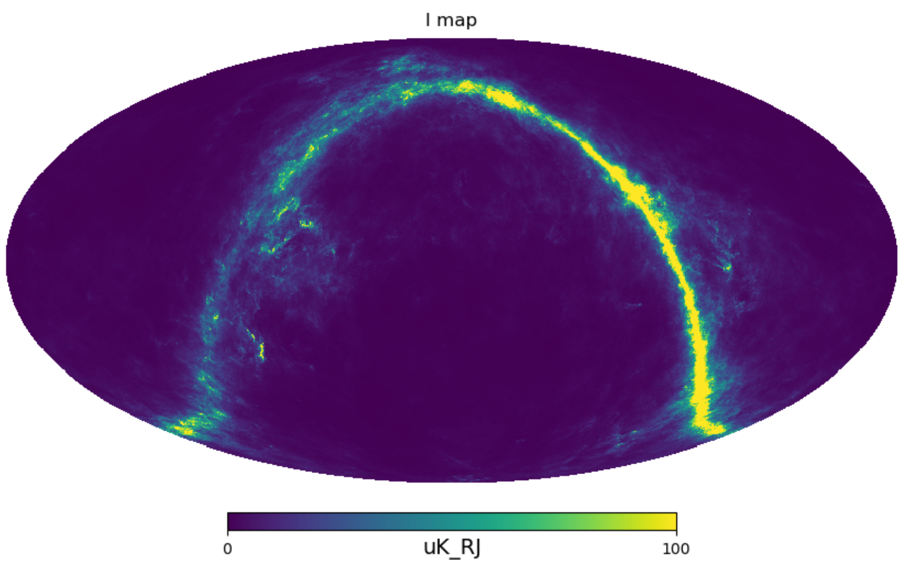

Figure 5: From the d12 foreground, this graph shows the foreground in celestial coordinates at a 30 GHz frequency.

Now we must generate a "mask" that will zero out all values excluding the region we want to evaluate.

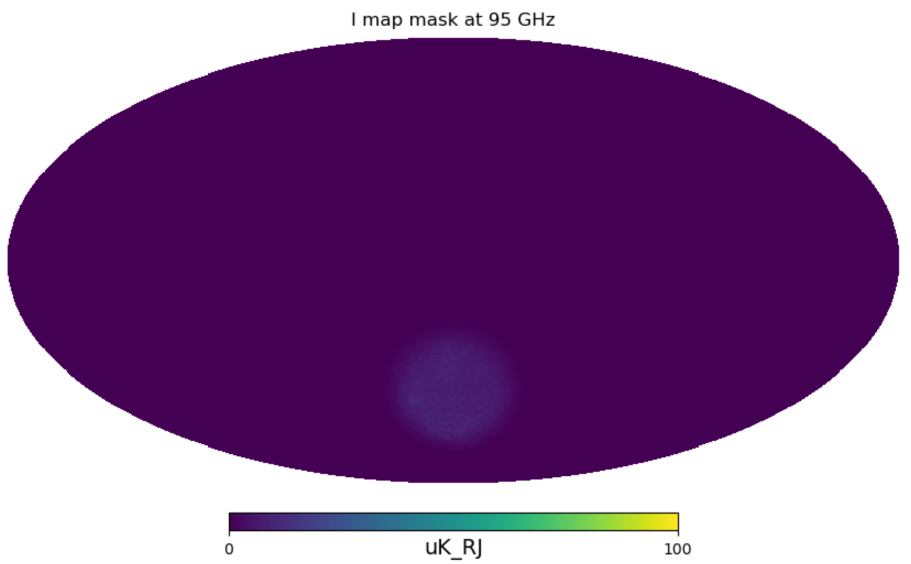

Figure 5: From the d9 foreground, this graph shows the masked foreground in celestial coordinates at a 95 GHz frequency.

3.3 Power Spectra & Cross Spectra

Next we want to do a Fourier transform of our maps in order to examine the difference in angular scale. From our above analysis, we will convert our maps from Temperature, Q polarization, and U polarization to Temperature, E-modes, and B-modes.

Now as we are examining a smaller area of our original mask, we know that the power must have decreased to some percent, which we will establish as 5%. In order to make up for this decrease in power, we will have to square the mask itself and divide by the number of pixels to obtain an adjustment fraction. This will be used in the next step.

Now to calculate the power spectra and the cross spectra we must apply a alm2cl function to our Fourier transformed maps. This function changes our temperature, E-modes, and B-modes maps into six spectra: TT, BB, EE, TB, TE, and EB maps. The TT, BB, and EE maps are similar to our previous maps, they are just squared and added together. Creating a higher amplitude. The other 3 are discussed more by Planck. But of these graphs, TB has a nontrivial correlation.TE and EB will turn out to be scattered noise which will be close to zero, so we do not worry about these maps. As we are looking at polarization by the galactic magnetic field, the main maps we will examine are BB.

Our next step will be to examine these BB slopes at varying frequencies. To do this we will establish a power-law to determine a theoretical fit of our BB graph.

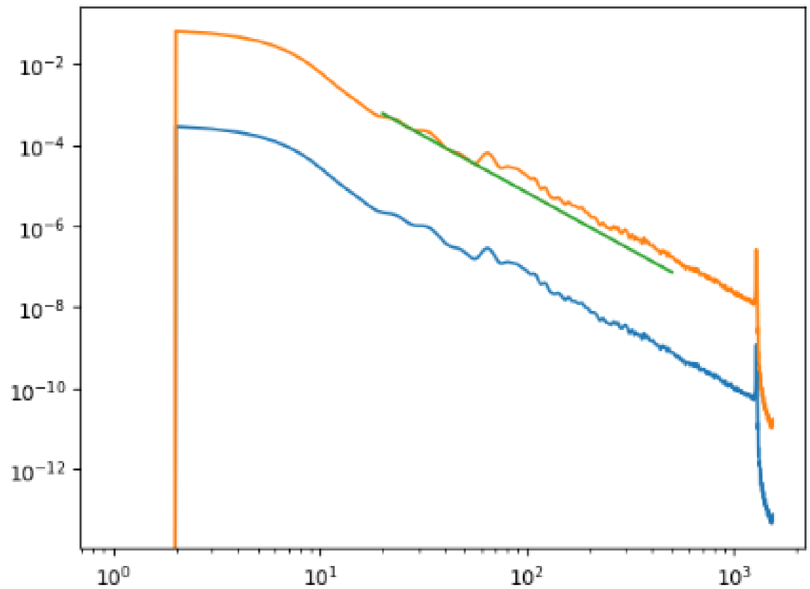

Figure 6: Shows the graph of BB at two frequencies. Orange is a frequency of 220GHz and Blue is a frequency of 30GHz. The green line is our power-law approximate

This fit will give us an approximate amplitude for the temperature. If we take these values for each frequency then we may combine them as a list in a loglog plot. By doing this we are examining the power spectra of the resulting data.

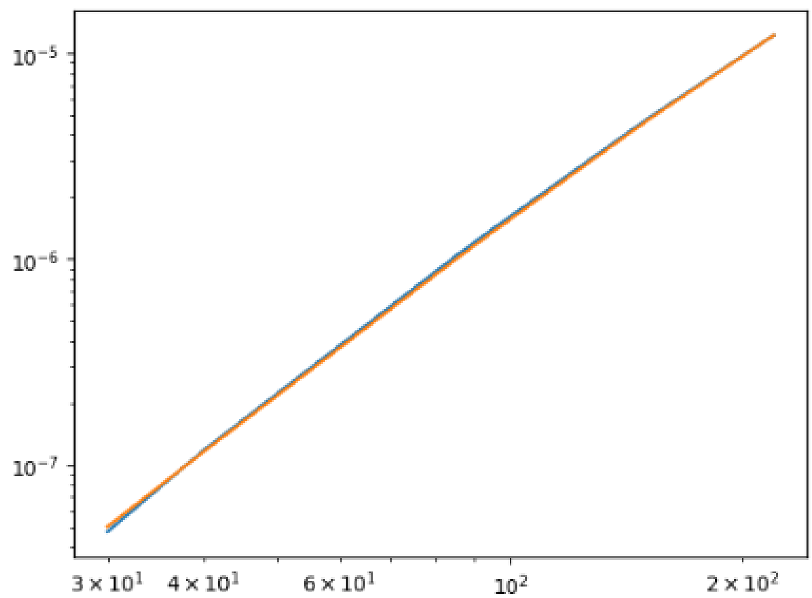

Figure 7: Shows the power law from the theoretical (Orange) and the actual map (Blue). Here our x-axis is the frequency and the y-axis is the temperature.

It should be noted that for each graph we obtain a different value for β. For d9 $\beta = 1.5$, for d10 $\beta = 1.7$, and for d12 $\beta = 1.5$. Our expected values of β lie around 1.5 and 1.6, so these are agreeable results.

3.3.1 Cross Spectra

Next we will examine the cross spectra of the maps. For this, we will be using our power-law values. Specifically for this, we want to multiply one frequency to another. This aligns with the idea of template cleaning. As there are 21 possible combinations we may create a function to calculate these points and then graph them along with our power law to see if there is good alignment between the Power spectra and cross spectra. If we observe good alignment then we may say that there is correlation between dust across frequency.

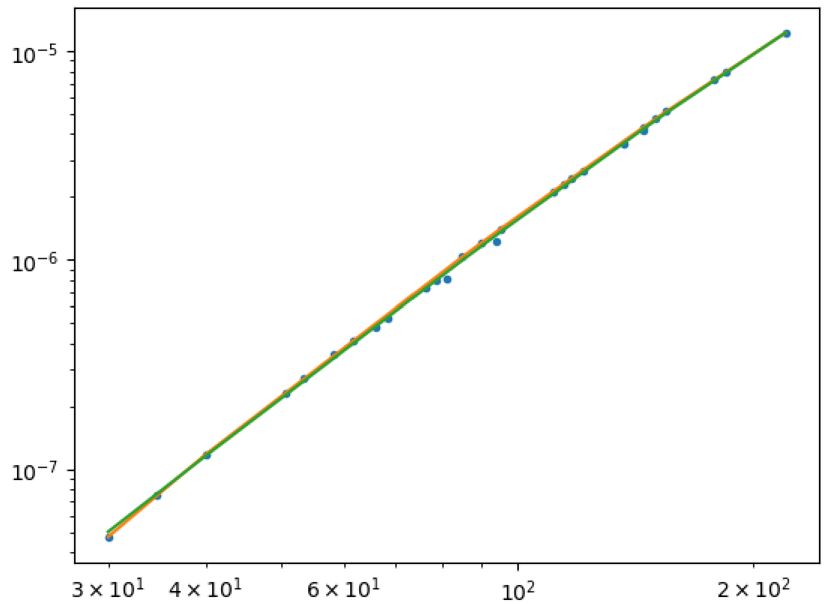

Figure 8: Here our x-axis is the frequency and the y-axis is the temperature. Our d9 foregrounds auto spectra and cross spectra. The blue points are our cross spectra values, the Green line is our theoretical power-law, and the Orange line is the actual auto spectra. Here we observe good alignment.

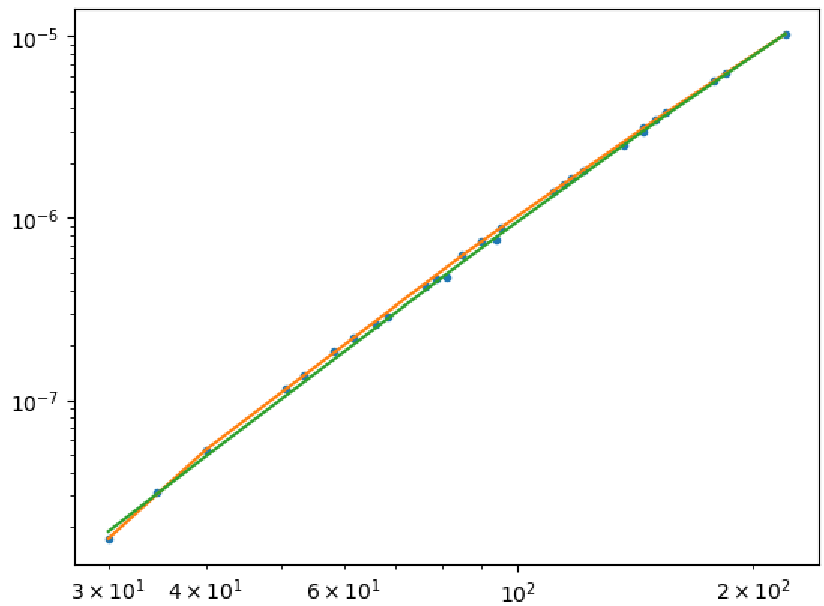

Figure 9: Here our x-axis is the frequency and the y-axis is the temperature. Our d10 foregrounds auto spectra and cross spectra. The blue points are our

cross spectra values, the Green line is our theoretical power-law, and the Orange line is the actual auto spectra. Here our alignment is not quite as good as in d9, but it is still acceptable.

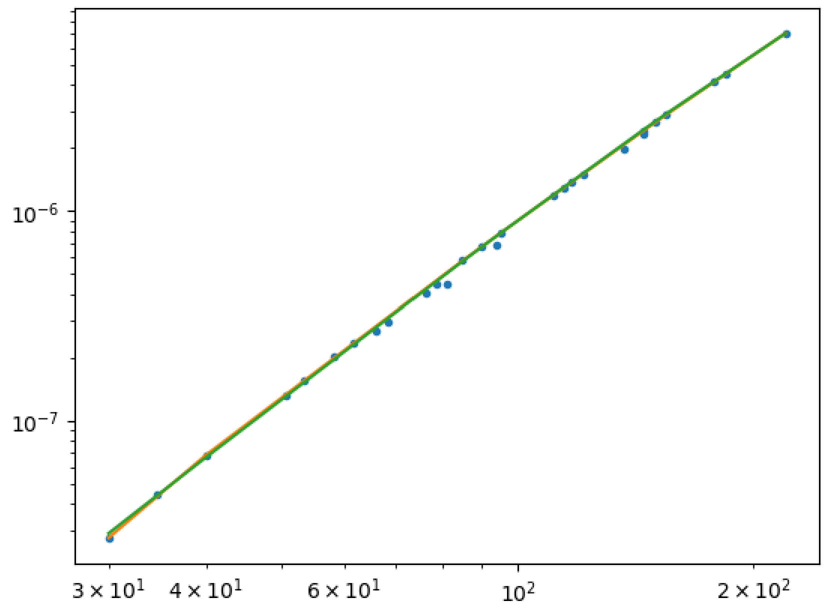

Figure 10: Here our x-axis is the frequency and the y-axis is the temperature. Our d12 foregrounds auto spectra and cross spectra. The blue points are our cross spectra values, the Green line is our theoretical power law, and the Orange line is the actual auto spectra. Here we observe good alignment outside of the cross spectra. While the cross spectra are still generally aligned, we do observe some frequencies where the cross spectra are below the expected.

From the above graphs, we are overall able to see a good alignment between the power spectra and the auto spectra. Whereas our more complex maps result in slightly less alignment, which is expected. Thus we may say that There is good correlation between dust across frequency.

4 Conclusion

We have discussed the significance of looking at the correlation between frequency and our foregrounds, specifically dust. As frequency is increased the temperature from dust is decreased. To this end, we generated 3 different foreground maps and applied various frequencies to explore their effects on each model. Using the polarization of these foregrounds we were able to model graphs for their projected BB spectrum. Finally, we were able to take this and explore the power law, including auto and cross-spectrum, for the dust model at varying frequencies. Which revealed a good correlation between the theorized power spectra and the cross spectra.

The biggest question this experiment raises has to do with the next steps toward the overall goal, which is examining the polarization of CMB. If we conduct a similar process but with synchrotron would we also be able to establish a correlation between frequency? If we are able to, then we would be able to 'remove' synchrotron in the same way we may remove dust, allowing us to view only the CMB.

5 References

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