Axioms of Quantum Mechanics — long version
(Underlined terms are linear algebra concepts whose definitions you need to know.)

Axioms:

I. The state of a system is a vector, $|\psi\rangle$, in a Hilbert space (a complex vector space with a positive definite inner product), and is normalized: $\langle \psi | \psi \rangle = 1$.

II. An observable (allowed measurement) is a choice of a hermitean operator, $\hat{M}$. By the spectral theorem, $\hat{M} = \sum_i \mu_i \hat{P}_i$, where $\mu_i$ are its eigenvalues and $\hat{P}_i$ are the orthogonal projection operators onto their corresponding eigenspaces.

III. The only possible outcomes of measuring $\hat{M}$ are one of its eigenvalues. I denote this outcome of this measurement by "$M = \mu_i$".

IV. The probability of observing a given possible outcome, $M = \mu_i$, of such a measurement is the norm-squared of the projection of the state onto the eigenspace of the eigenvalue $\mu_i$. In formulas, this is

$$P(M=\mu_i) = \langle \psi | \hat{P}_i | \psi \rangle = \left\| \hat{P}_i | \psi \rangle \right\|^2.$$ 

V. Once we observe the outcome $M = \mu_i$, the state changes as a result of the measurement from its state $|\psi\rangle$ immediately before the measurement, to a new state $|\psi'\rangle$ immediately after the measurement, given by its normalized projection onto the eigenspace corresponding to the observed eigenvalue. In formulas, this is

$$|\psi\rangle \xrightarrow{\text{meas. } M=\mu_i} |\psi'\rangle = \frac{\hat{P}_i |\psi\rangle}{\sqrt{P(M=\mu_i)}} = \frac{\hat{P}_i |\psi\rangle}{\left\| \hat{P}_i | \psi \rangle \right\|}.$$ 

VI. The time evolution of the state of an isolated system (i.e., when it is not being measured or otherwise interacting with the external world) is given by $|\psi(t)\rangle = \hat{U}(t) |\psi(0)\rangle$, where the unitary time evolution operator is given by $\hat{U}(t) = \exp\{-it\hat{H}/\hbar\}$ where $\hat{H}$ is the hermitean energy operator (also known as the Hamiltonian operator).