Centroid and Moment of Inertia

Moment of Inertia  \( I = \int y^2 \, dA \)

Numerically large when material distributed away from the neutral axis (above or below, distance is squared)

Same shape different orientation:

\( I_x \) large \( \Rightarrow \) resists bending
\( I_x \) smaller

\[ r_y = 7.21 \text{ mm} \]

\[ r_x = 14.43 \text{ mm} \]

Note: symmetric about \( y \)

\[ \bar{y} = \frac{\Sigma (A \cdot y)}{\Sigma A} \]

\[ I_{\text{total}} = \Sigma I + \Sigma (A \cdot d^2) \quad I_x = r_x^2 \cdot A \]

\[ I_{\text{rectangle}} = \frac{b \cdot h^3}{12} \quad I_{\text{circle}} = \frac{\pi d^4}{64} \]

b = parallel dimension

Review from Statics
Divide shape into sections and number
Select datum (0,0)
Make table,
Row = section#
Column = terms in equations
\( y \) = distance from datum to section centroid
\( d \) = distance from shape centroid to section centroid
\( \left| y_c - y \right| \)

Radius of gyration = average distance
\( r_x = \sqrt{\frac{I_x}{A}} \)

<table>
<thead>
<tr>
<th>( A(\text{mm}^2) )</th>
<th>( y(\text{mm}) )</th>
<th>( A\cdot y(\text{mm}^3) )</th>
<th>( I_x(\text{mm}^4) )</th>
<th>( d_y(\text{mm}) )</th>
<th>( A\cdot d_y^2(\text{mm}^6) )</th>
<th>( x(\text{mm}) )</th>
<th>( A\cdot x(\text{mm}^3) )</th>
<th>( I_y(\text{mm}^4) )</th>
<th>( d_x(\text{mm}) )</th>
<th>( A\cdot d_x^2(\text{mm}^6) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5'15</td>
<td>75</td>
<td>2812.5</td>
<td>15.015</td>
<td>156.25</td>
<td>-17.5</td>
<td>22968.75</td>
<td>10</td>
<td>1406.25</td>
<td>-10</td>
</tr>
<tr>
<td>2</td>
<td>40'5</td>
<td>200</td>
<td>4000</td>
<td>400/12</td>
<td>26666.7</td>
<td>0.00</td>
<td>0</td>
<td>0</td>
<td>416.67</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>15'5</td>
<td>75</td>
<td>187.5</td>
<td>156.25</td>
<td>17.50</td>
<td>22968.75</td>
<td>10</td>
<td>1406.25</td>
<td>-10</td>
<td>7500</td>
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<tr>
<td></td>
<td></td>
<td>350</td>
<td>7000</td>
<td>26979.2</td>
<td>45937.5</td>
<td></td>
<td></td>
<td></td>
<td>3229.17</td>
<td>15000</td>
</tr>
</tbody>
</table>

\( \bar{y} = 20 \text{ mm} \)

\( I_x = 72916 \text{ mm}^4 \)

\( x_c = 0 \text{ mm} \)

\( \bar{y} = 18229 \text{ mm}^4 \)
1 Write Fundamental Equations

2 Write differential equations

3 Substitute to get \( \sigma_{\text{max}} \) as \( f(M) \)

4 Separate variables and integrate

5 Define \( I = \int y^2 \, dA \) Solve for \( \sigma_{\text{max}} \)

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1 Bending Equation \( \sigma = \frac{kMc}{I} \) or \( \frac{kM}{S} \)

k = 1 for no stress concentration

2 Find A & B using symmetry

3 Sketch V diagram

5 Use largest M value in \( \sigma = \frac{kMc}{I} \) or \( \frac{kM}{S} \)
A wood platform is to be made of standard plywood and finished lumber using the cross section shown in Figure P7–18(a). Would the platform be safe if four men, weighing 250 lb each, were to stand 2 ft apart, as shown in Figure P7–18(b)? Consider only bending stresses (see Chapter 8 for shear stresses).

1. Find A & B using symmetry
2. Draw shear diagram
3. Draw moment diagram
Bending Stress - Design
Problem 7-18 Continued

4 Prepare cross-section diagram to find $I_x$
   a) put x,y origin location 0,0 on diagram
   b) label sections of diagram

5 Make table for $I_x$ calculation using section numbers (rows) and equation components (columns)

\[ y_c = \frac{\Sigma(A\cdot y)}{\Sigma A} \quad I_{x\ total} = \Sigma I_x + \Sigma(A \cdot d_y^2) \quad I_{x\ rectangle} = \frac{b+h^3}{12} \quad d_y = y - y_c \]

<table>
<thead>
<tr>
<th>A(in^2)</th>
<th>y(in)</th>
<th>A*y(in^3)</th>
<th>I_x(in^4)</th>
<th>d_y(in)</th>
<th>A*d_y^2(in^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2\times3.5\times1.5</td>
<td>10.5 0</td>
<td>0</td>
<td>10.5</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>2 2\times0.5\times24 (+/-)3.5/2+0.25</td>
<td>24 0</td>
<td>0</td>
<td>0.5</td>
<td>2.00</td>
<td>96</td>
</tr>
</tbody>
</table>

yc 0
lx 107.2188

6 \[ \sigma = \frac{kMc}{l} \]

7 Compare to allowable stress for wood. Safe?
A bench for football players is to carry the load shown in Figure P7–64 approximating the case when 10 players, each weighing 300 lb, sit close together, each taking 18 in of the length of the bench. If the cross section of the bench is made as shown in Figure P7–64, would it be safe for bending stress? The wood is No. 2 grade hemlock.

**FIGURE P7–64** Bench and load for Problems 7–64, 7–65, 7–66, and 7–67.

**Material Properties** pg 719
Compute the stress due to bending at the fulcrum, 20 in from the pivot and at each of the holes in the bar. The diameter of each hole is 0.75 in.

1. $\Sigma M_A = 0$ to find $B$

2. $\Sigma F_{y'} = 0$ to find $A$

3. Sketch $V$ to $x'$ plot

4. Sketch $M$ to $x'$ plot

5. Find $\sigma_B = \frac{kM}{S}$ \hspace{1cm} (k=1)

6. Find worst case hole.

$\sigma_H = \frac{kM}{S}$

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