An Upper Bound

Problem 91-8, *by* PHILIP KORMAN (University of Cincinnati).

For all $t \geqq 1$ the function $F(t)$ is continuous, nonnegative, and satisfies

$$tF(t) \leqq c + \int_1^t F(t)\ dt \tag{1}$$

where c is a positive constant. Show that $F(t) \leqq c$ for all $t \geqq 1$.

Problem 91-8 (*Quickie*).

Solution.

From (1),

$$F(t) \Big/ \left\{ C + \int_1^t F(t)\,dt \right\} \leqq 1/t.$$

Integrating from 1 to t and taking antilogs, gives

$$c + \int_1^t F(t)\,dt \leqq ct.$$

Then using (1) again, we get the desired result.