

**An Introduction to Partial Differential Equations.** By M. Renardy and R. C. Rogers. Springer-Verlag, New York, 1993. 428 pp., \$42.00, cloth. ISBN 0-387-97952-2.

This is a textbook on partial differential equations, aimed at beginning graduate students. In addition to linear equations, it presents quite a number of results on nonlinear equations. The material ranges from separation of variables to applications of pseudomonotone operators to quasilinear PDEs. There is a large body of background material from functional analysis presented in detail. In particular, there is a full chapter on operator theory, a chapter on distribution theory and the Fourier transform, and about thirty pages on Banach and Hilbert spaces.

There are many things that I liked in this book. For example, there is a very nice presentation of the theorem of B. Gidas, W.-M. Ni, and L. Nirenberg on symmetry of solutions of semilinear elliptic equations in a ball. There is also a very concise presentation of both interior and boundary regularity theory for second order linear elliptic equations. There are nice touches throughout the book. For example, the statement of the Hopf boundary lemma includes a useful case, usually skipped in other books (when the solution vanishes). There are many (doable) exercises.

I tried this book in a graduate PDE course. It appears that the book is more suitable as a supplement, rather than as a main text. This brings me to the quibbles part. While the presentation is concise (which I liked), sometimes it is too hurried. For example, the Poisson formula for the Dirichlet problem on a ball appears from nowhere on p. 112. The missing background material (involving the Green's function) would not only provide the motivation, but is also interesting in its own right (see [2]). Another example is the definition of characteristic surfaces. Recall that a point is called noncharacteristic if, using the initial data and the equation itself, one can compute all derivatives of the solution at that point. In such a case one can write down the formal Taylor series

for the solution, and try to prove its convergence. The definition given on p. 41, while equivalent to the one above, is completely unmotivated. In H. Lewy's example of a PDE with no solution, a proof is started, but not completed. I think that in an introductory textbook it is better to either give complete proofs or skip them altogether. By the way, I wonder if it is worth presenting the proof of Lewy's theorem in PDE textbooks, since both the result and the method of its proof seem to be so different from most current research (see also L. Nirenberg [4] and F. John [3] for a different example and a discussion).

Despite the above reservations, I find this book to be a most welcome addition to the literature, with a nice presentation of many results, including some that are hard to find in other books. There are remarkably few misprints, and the quality of production (by Springer) is high. I think this book is likely to become one of the main introductory PDE textbooks, along with the books by F. John [3], L. Evans [1], and D. Gilbarg and N. S. Trudinger [2].

It seems that publishers of mathematics regularly read reviews like this. At least quotes from such reviews (often, with many dots added) regularly appear in their promotional literature. I hope that Springer will find it possible to reissue the classical book by Gilbarg and Trudinger [2], and keep it in print.

#### REFERENCES

- [1] L. C. EVANS, *Partial Differential Equations*, Berkeley Mathematics Lecture Notes, Vol. 3, A and B, 1994.
- [2] D. GILBARG AND N. S. TRUDINGER, *Elliptic Partial Differential Equations of Second Order*, 2nd ed., Springer-Verlag, New York, 1983.
- [3] F. JOHN, *Partial Differential Equations*, 4th ed. Springer-Verlag, New York, 1982.
- [4] L. NIRENBERG, *Lectures on Linear Partial Differential Equations*, Regional Conf. Ser. in Math. 17, AMS, Providence, RI, 1973.

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