A Pedagogically Correct Way to Square it Out

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Generations of students had to sit through the following dull, boring and geeky derivation

$$(a+b)^2 = aa + ab + ba + bb = a^2 + 2ab + b^2.$$

The modern pedagogical thought has clearly identified the problems with this "plug-and-chug" approach: lack of intuitive reasoning, absence of graphical motivation, and no connections with the Real World. Moreover, the powers of modern technology are left untapped. The purpose of this note is to move toward remedying this woeful state of affairs, and to show the pedagogically correct and socially responsible way for the new millennium.

A lumberjack has 5 nails and 3 oz. of ketchup, which she uses to lubricate the nails. Each nail is 2.6 inches long. She also has 7 smaller nails, 1.7 inches long. After lubricating the nails, the worker makes a square box placing one long and two shorter nails on each side. Figure out the area of the box.

After each group of students will thouroughly discuss the problem, the teacher may start guiding them toward a solution. If you imagine the box cut up in four pieces, like on the drawing that I made, then there are two square boxes (one bigger than the other) and two rectangular ones (which look about the same). The side of the bigger square box is $2 \times 1.7 = 3.4$ inches, and the side of the smaller square box is 2.6 inches. The sides of the rectangular boxes will be computed, when you solve the exercises 1, and 2 respectively. You then have for the area of the big box

$$3.4 \times 3.4 + 3.4 \times 2.6 + 2.6 \times 3.4 + 2.6 \times 2.6 = 36,$$

which is of course the same as $(3.4 + 2.6)^2$. (Do you see that?) In the next lecture the teacher should go over this material, and begin introducing letters (like a, b, etc.) to denote the quantities involved (like nails and ketchup).

Helpful Drawing 1.