Axioms of Quantum Mechanics — long version

(Underlined terms are linear algebra concepts whose definitions you need to know.)

Axioms:

I. The *state* of a system is a vector, $|\psi\rangle$, in a Hilbert space (a complex vector space with a positive definite inner product), and is normalized: $\langle\psi|\psi\rangle = 1$.

II. An *observable* (allowed measurement) is a choice of a hermitean operator, $\widehat{M}$. By the spectral theorem, $\widehat{M} = \sum_i \mu_i \widehat{P}_i$, where μ_i are its eigenvalues and $\widehat{P}_i$ are the orthogonal projection operators onto their corresponding eigenspaces.

III. The only possible *outcomes* of measuring $\widehat{M}$ are one of its eigenvalues. I denote this outcome of this measurement by "$M = \mu_i$".

IV. The *probability* of observing a given possible outcome, $M = \mu_i$, of such a measurement is the norm-squared of the projection of the state onto the eigenspace of the eigenvalue μ_i. In formulas, this is

$$\mathcal{P}(M{=}\mu_i) = \langle\psi|\widehat{P}_i|\psi\rangle = \left\| \widehat{P}_i|\psi\rangle \right\|^2 .$$

V. Once we observe the outcome $M = \mu_i$, the *state changes as a result of the measurement* from its state $|\psi\rangle$ immediately before the measurement, to a new state $|\psi'\rangle$ immediately after the measurement, given by its normalized projection onto the eigenspace corresponding to the observed eigenvalue. In formulas, this is

$$|\psi\rangle \xrightarrow{\text{meas. } M=\mu_i} |\psi'\rangle = \frac{\widehat{P}_i|\psi\rangle}{\sqrt{\mathcal{P}(M{=}\mu_i)}} = \frac{\widehat{P}_i|\psi\rangle}{\left\|\widehat{P}_i|\psi\rangle\right\|}.$$

VI. The *time evolution* of the state of an isolated system (i.e., when it is not being measured or otherwise interacting with the external world) is given by $|\psi(t)\rangle = \widehat{U}(t)|\psi(0)\rangle$, where the unitary time evolution operator is given by $\widehat{U}(t) = \exp\{-it\widehat{H}/\hbar\}$ where $\widehat{H}$ is the hermitean energy operator (also known as the Hamiltonian operator).