Electrostatics of linear media and vacuum magnetostatics

Electrostatics

$\vec{F}(\vec{r}) = q\vec{E}(\vec{r})$	Lorentz force law
$\vec{E}(\vec{r}) = \dfrac{1}{4\pi\epsilon}\displaystyle\int d\tau' \, \dfrac{\rho_f(\vec{r}\,')\hat{\mathscr{r}}}{\mathscr{r}^2}$	Coulomb law
$\vec{\nabla}\cdot\vec{E} = \dfrac{1}{\epsilon}\rho_f \quad \Leftrightarrow \quad \displaystyle\oint_S d\vec{a}\cdot\vec{E} = \dfrac{1}{\epsilon}Q_{f,\text{enc}}$	Gauss law
$\vec{\nabla}\times\vec{E} = 0 \quad \Rightarrow \quad \vec{E} = -\vec{\nabla}V$	def'n electric potential
$\nabla^2 V = -\dfrac{1}{\epsilon}\rho_f$	Poisson eqn
$V(\vec{r}) = \dfrac{1}{4\pi\epsilon}\displaystyle\int d\tau' \dfrac{\rho(\vec{r}\,')}{\mathscr{r}}$	Poisson eqn sol'n
$\epsilon = \epsilon_0(1+\chi_e)$	permittivity and susceptibility
$\vec{D} = \epsilon\vec{E} \qquad \vec{P} = (\epsilon-\epsilon_0)\vec{E}$	displacement and polarization
$\rho_b = \dfrac{\epsilon_0-\epsilon}{\epsilon}\rho_f$	bound charge density
$V_{\text{out}} = V_{\text{in}} \qquad \epsilon_{\text{out}}\dfrac{\partial V_{\text{out}}}{\partial n} - \epsilon_{\text{in}}\dfrac{\partial V_{\text{in}}}{\partial n} = -\sigma_f$	dielectric boundary conditions
$\sigma_b = (\vec{P}_{\text{in}} - \vec{P}_{\text{out}})\cdot\hat{n}$	bound surface charge density
$V_{\text{out}} = V_{\text{in}} \qquad \epsilon_{\text{out}}\dfrac{\partial V_{\text{out}}}{\partial n} = -\sigma_f$	conductor boundary conditions
$V_{\text{in}} = \text{constant}$	conductor property
$V(\vec{r}) = \displaystyle\sum_{\ell=0}^{\infty}(A_\ell r^\ell + B_\ell r^{-\ell-1})P_\ell(\cos\theta)$	sol'n Laplace eqn w axial symmetry
$V(\vec{r}) = \dfrac{1}{4\pi\epsilon}\displaystyle\sum_{\ell=0}^{\infty}\dfrac{M_\ell}{r^{\ell+1}} \qquad M_\ell = \displaystyle\int d\tau'(r')^\ell P_\ell(\cos\theta')\rho(\vec{r}\,')$	multipole expansion
$M_0 = \text{total charge}$	electric monopole moment
$M_1 = \vec{p}\cdot\hat{r} \qquad \vec{p} = \displaystyle\int d\tau' \, \vec{r}\,' \, \rho(\vec{r}\,')$	electric dipole moment
$V_{\text{dip}}(\vec{r}) = \dfrac{\vec{p}\cdot\hat{r}}{4\pi\epsilon r^2} \qquad \vec{E}_{\text{dip}}(\vec{r}) = \dfrac{3(\vec{p}\cdot\hat{r})\hat{r}-\vec{p}}{4\pi\epsilon r^3}$	dipole fields

Magnetostatics

$\vec{F}(\vec{r}) = q\vec{v}\times\vec{B}(\vec{r})$	Lorentz force law
$\vec{B}(\vec{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int d\tau'\, \dfrac{\vec{J}(\vec{r}\,')\times\hat{\mathscr{r}}}{\mathscr{r}^2}$	Biot-Savart law
$\vec{\nabla}\cdot\vec{J} = 0 \quad\Rightarrow\quad I = \text{constant}$	continuity eqn for steady current
$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \quad\Leftrightarrow\quad \displaystyle\oint_C d\vec{\ell}\cdot\vec{B} = \mu_0 I_{\text{enc}}$	Ampere law
$\vec{\nabla}\cdot\vec{B} = 0 \quad\Rightarrow\quad \vec{B} = \vec{\nabla}\times\vec{A}$	def'n vector potential
$\vec{\nabla}\cdot\vec{A} = 0$	Coulomb gauge
$\nabla^2\vec{A} = -\mu_0\vec{J}$	Poisson eqn
$\vec{A}(\vec{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int d\tau'\,\dfrac{\vec{J}(\vec{r}\,')}{\mathscr{r}}$	Poisson eqn sol'n
$\mu_0\epsilon_0 = c^{-2}$	permeability and speed of light
$\vec{B}_{\text{out}} - \vec{B}_{\text{in}} = \mu_0\vec{K}\times\hat{n}$	surface current boundary condition
$\vec{A}_{\text{out}} = \vec{A}_{\text{in}} \qquad \dfrac{\partial\vec{A}_{\text{out}}}{\partial n} - \dfrac{\partial\vec{A}_{\text{in}}}{\partial n} = -\mu_0\vec{K}$	surface current boundary conditions
$\vec{A}(\vec{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\sum_{\ell=1}^{\infty}\dfrac{\vec{M}_\ell}{r^{\ell+1}} \qquad \vec{M}_\ell = \displaystyle\int d\tau'(r')^\ell P_\ell(\cos\theta')\vec{J}(\vec{r}\,')$	multipole expansion
$\vec{M}_1 = -\vec{m}\times\hat{r} \qquad \vec{m} = \dfrac{1}{2}\displaystyle\int d\tau'\,\vec{r}\,'\times\vec{J}(\vec{r}\,')$	magnetic dipole moment
$\vec{A}_{\text{dip}}(\vec{r}) = \dfrac{\mu_0 m\sin\theta\hat{\phi}}{4\pi r^2} \qquad \vec{B}_{\text{dip}}(\vec{r}) = \dfrac{\mu_0\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]}{4\pi r^3}$	dipole fields ($\vec{m} = m\hat{z}$)